# Offline Size and Online Scale: A Tale of Two Platforms

Huihui Li, Kai Li*

October 9, 2022

**Abstract**

The online market can be categorized into two platforms: the reseller and the marketplace. Motivated by the fact that the relative scale of reseller to marketplace is larger in developed countries (e.g., the United States) than in developing countries (e.g., China), our study proposes a novel explanation characterized by different pricing mechanisms for this online scale difference: we attribute this online scale difference to an offline determinant, the firm size distribution. Decentralized pricing, commonly adopted by a marketplace to attract firms to sell from it, is more favorable to smaller firms compared with the centralized pricing set by a reseller. Thus, the relative scale of the marketplace to the reseller is larger in developing countries, given that the offline firm size distribution in developing countries is skewed towards small firms compared with developed countries.

**Keywords:** Online Platform Scale, Firm Size Distribution, Reseller, Marketplace

**JEL Classification: L60, L81**

*Huihui Li: School of Economics and WISE, Xiamen University. Email: huihui.li@outlook.com; Kai Li: Department of International Economics and Business, School of Economics and WISE, Xiamen University, China. Email: likai501@gmail.com. Authors contribute equally. The project is supported by the Fundamental Research Funds for the Central Universities (No. 2072021047) and the Basic Scientific Center Project of NSF in China (No. 71988101), Xiamen University. All errors are ours.

# 1 Introduction

Online market platforms cannot live without offline firms. In general, there are two common platforms: the reseller and marketplace. A reseller purchases products from suppliers (firms) and sells them to customers from its own warehouses. By contrast, a marketplace offers a place through which firms sell directly to customers. The scale of a platform sheds light on the degree to which an online market is accessible. Access to a platform empowers firms to reach a broader range of customers, enabling them to enjoy more products. Over the past two decades, the scale of platforms in the online market has expanded rapidly. As of July 2022, Amazon's market capitalization reached 1.115 trillion USD, whereas that of Alibaba was 310.3 billion USD. Their exceptional performance in the capital market is highly dependent on their rapidly growing scale in the product market.

In the United States and China, each representing developed and developing countries, world-leading e-commerce platforms have transformed the retail landscape. Among these, Amazon, eBay, Jingdong (JD), and Alibaba are the top e-commerce giants in the United States and China, respectively. Both Amazon and JD are considered resellers given that at least half of their total sales are made by themselves.[1] Their counterparts, eBay and Alibaba, are organized in the form of marketplaces where third-party sellers contribute to all sales. There are multiple platforms in both countries, but it is sufficient to consider the largest platform and disregard the small ones as multi-homing of offline firms is permitted and the largest platform may carry the most offline firms.

Cross-country comparisons among these platforms are subject to stark differences in fundamentals in the United States and China such as the number of firms, population size, and GDP per capita. Instead, the within-country comparisons of the two platforms indicate that the relative scale of the reseller to the marketplace is larger in the United States than in China. Alternatively, the reseller tends to be larger than the marketplace in the United States than in China. Is this phenomenon accidentally or intrinsically determined? Our study offers a novel explanation for this cross-country differences in the relative scale between the two platforms, linking it to an offline fundamental that notably differs between developed and developing countries—the firm size distribution.

Both Amazon and eBay initiated their online businesses around 1995, and JD and Alibaba began online sales in 2003. The synchronization of platform development in the United States and China precludes the timing difference in their establishment where one platform may develop

[1] Third-party sales in Amazon have increased over time and remained around 50% of its total sales since 2016. In this regard, Amazon could be considered either a marketplace or a reseller.

earlier as a possible explanation for the current scale difference. Although platforms in the online market reduce the searching cost of customers (Bakos, 1997) substantially expanding the demand for offline firms, not all firms run a business online, because it might not be profitable to do so.

One significant pattern for firms in developing countries such as China is that the proportion of small firms is high, making the firm size distribution skewed towards small firms compared with developed countries such as the United States. Cross-country evidence regarding firm size distribution and development stage has been extensively investigated (Hsieh and Olken, 2014; Tybout, 2000). It was found that it is difficult for firms in developing countries to grow large. Firms in the United States and China are no exception to this.

To further explore how offline firm size distribution shapes the platform scale difference, we first established that the relative scale of the reseller to the marketplace is larger in the United States than in China, and further found the pattern holds beyond the United States and China. Although the two platforms are in contrast in many ways, such as product categories and logistics, by singling out the pricing mechanism as the key factor in distinguishing the reseller and marketplace, we developed a tractable model to explain how the relative scale of online platforms is affected by the distribution of firm sizes under different pricing mechanisms. Knowing the offline firm size distribution, but not the individual supplier's cost, a reseller purchases from firms and sets centralized prices on its own because engaging in purchases makes it easy for the reseller to aggregate price and product information. In contrast, the marketplace decentralizes pricing to individual firms and allows them to manage products themselves.[2] Centralized pricing is more favorable for large firms than the decentralized pricing adopted by the marketplace.

We decomposed the overall effect of firm size distribution on the scale of platforms into two margins: one is connected to the revenue of each firm and the other is attributed to the proportion of offline firms going online. Under intuitive conditions, the model generates a result consistent with the empirical observation of the relative scale of platforms. That is, if the firm size distribution skews more towards small firms, the relative scale of the reseller to the marketplace will be smaller. Such a result holds when the marginal cost of production varies with the size of the firm like the demand. We also found evidence that supports our model assumption. Our study ignores the reverse effect that online platforms also encourage newborn firms (Lieber and Syverson, 2012), and focuses on the offline determinants of the online landscape.

While online platforms flatten the business world, online landscapes are shaped by offline fundamentals. However, this study does not rule out other determinants. Instead, it provides a novel explanation that has yet to be explored to account for the scale difference in platforms. This

[2] In reality, a reseller employs various price discrimination tactics among customers, which is not the interest of this study.

study supplements the vast literature studying interactions between online businesses and offline firms. Additionally, it contributes to the consequence of a long-lasting topic regarding economic development—the firm size distribution—where various causes are explored.

The remainder of this paper is organized as follows: Section 2 reviews the literature. Section 3 presents the motivation for relative scale differences. Section 4 provides a model to explain the linkage between offline firm size distribution and the relative scale of the platforms. Section 5 extends the model to allow marginal cost to vary with the firm size. The final section concludes this paper.

## 2 Literature

This study is relevant to two strands of literature. While the first strand of literature investigates firm size distribution, the second strand discusses various determinants shaping platforms in the online market.

Firm size distribution is a long-lasting topic in economic development. Cross-country comparisons highlight that firm size distribution in developing countries is in stark contrast to that of developed countries (Bento and Restuccia, 2021). It is well documented that many firms in developing countries remain small and cannot grow large, leading to the phenomenon that medium and large firms are absent (Hsieh and Olken, 2014; Tybout, 2000). Most studies explore the various determinants of firm size distribution, such as technology (Poschke, 2018), policy distortion (Bartelsman et al., 2013; Guner et al., 2008), or misallocation (Bento and Restuccia, 2017; Hsieh and Klenow, 2009). In addition, cross-country evidence shows that firm size, on average, is larger and significantly more dispersed in developed countries (Alfaro et al., 2009). Our study built on this consensus about firm size distribution and explored its effects on the online platform scale.

The second strand of literature investigates various determinants that shape platforms in the online market, especially the reseller and marketplace (Rochet and Tirole, 2006; Rysman, 2009). While the marketplace allows suppliers to sell directly to customers via a platform, the reseller, usually in the hybrid mode, mainly resells the products that they purchase from suppliers to customers. These two types of platforms differ in many aspects, such as product categories (Brynjolfsson and Smith, 2000; Hagiu and Wright, 2015), control rights (Boudreau, 2017; Hagiu, 2007), channel structures (Abhishek et al., 2016; Armstrong, 2006), indirect network effects (Caillaud and Jullien, 2003; Hagiu and Lee, 2011), and pricing mechanisms (Einav et al., 2016; Farronato, 2018; Weyl, 2010). Among these micro-level characteristics, we highlight the pricing mechanism and its interaction with an offline aggregate-level characteristic—firm size distribution. On the one hand, decentralized pricing is favorable when information is dispersed among buyers and sellers and eliciting that information is too costly for the platform. On the other hand, centralized pricing is preferable when the platform can aggregate relevant information cheaply. Despite complex pricing mechanisms, a reseller such as Amazon or JD usually adopts centralized pricing. In contrast, marketplaces such as Alibaba and eBay decentralize pricing to individual sellers. Through the pricing mechanism, our study links the offline firm size distribution with the online platform scale, exploring the aggregate-level determinant that shapes the online landscape.

In addition, this study is relevant to the literature on the interactions between online businesses and offline firms (Brown and Goolsbee, 2002; Goldmanis et al., 2010; Jin and Kato, 2007). Admittedly, no single characteristic can sufficiently account for the scale difference of platforms that have evolved over the last two decades, and neither is this study able to do.

## 3 Motivation

To establish the pattern of the relative scale of the reseller to the marketplace regarding offline firm size distribution, we used the development level as a proxy for firm size distribution, presenting evidence from the United States and China, as well as cross-country evidence. As the two largest economies, the United States and China are also representatives of developed and developing countries, respectively.

The gross merchandise value (GMV) is a commonly used variable to measure the scale of e-commerce platforms.[3] It should be noted that the GMV measures the total value of items sold, and a large GMV does not necessarily imply high profit because of the different profit margins of the two platforms.

Amazon and JD are hybrids that also provide a marketplace for third-party sellers, but at least half of their sales come from warehouses. To compare the relative scale, we constructed two indices, $r$ and $r^*$:[4]

$$r = \frac{S_R}{S_M}, \qquad r^* = \frac{(1-\eta)S_R}{\eta S_R + S_M},$$

where $\eta$ denotes third-party share of sales. $S_R$ and $S_M$ denote the scales of the reseller and marketplace, respectively. While $r$ is a generous measure that directly compares the scale, $r^*$ is a parsimonious measure that excludes the third-party share. Table 1 shows that the relative scale of the reseller to the marketplace is larger in the United States than in China. The same pattern is also present in 2014 and 2015, suggesting a consistent pattern over time, which is shown in Appendix

[3]Statistics are mainly from https://www.marketplacepulse.com and https://www.statista.com.

[4]For example, the relative scale of the reseller to the marketplace in the United States measured in GMV are $r = 335/90.2 = 3.71$ and $r^* = 335 \times (1 - 0.53)/(335 \times 0.53 + 90.2) = 0.59$.

A.1.

Table 1: The Scale of Platforms: United States and China (2019)

| | United States | | China | |
|---|---|---|---|---|
| | Reseller (Amazon) | Marketplace (eBay) | Reseller (JD) | Marketplace (Alibaba) |
| Third-party share ($\eta$) | 53% | 100% | 55.4% | 100% |
| GMV (billion USD) | 335 | 90.2 | 297.9 | 818 |
| $r$ | 3.71 | | 0.36 | |
| $r^*$ | 0.59 | | 0.14 | |

In addition to evidence from the United States and China, representatives of both developed and developing countries, cross-country evidence was also aligned with the above pattern. We collected the scales of the two largest e-commerce platforms (marketplace and reseller), measured by the number of visits per month[5] for some large economies. Interestingly, the two largest platforms are organized as a reseller and marketplace, respectively, as shown in Appendix A.1.

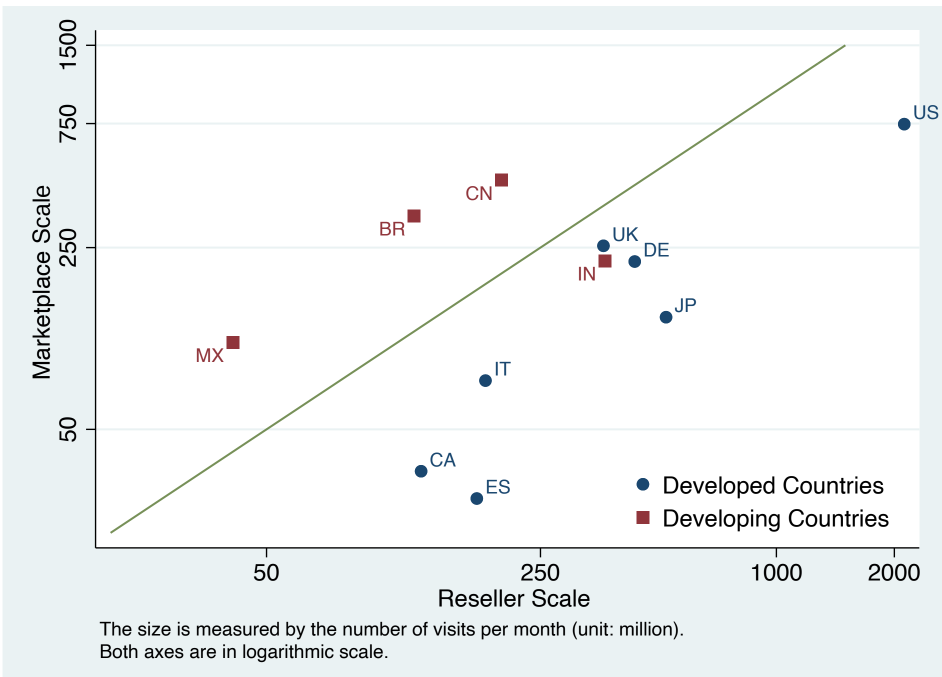


Figure 1: Size of E-commerce Giants: Cross-country Evidence (2019)

Countries were divided into two groups: developing and developed. Figure 1 plots the relative scale of the marketplace to the reseller measured by the number of visits per month. In developed countries, the relative scale of the marketplace to the reseller lies below the diagonal line, suggest-

[5]Statistics are mainly from https://zh.disfold.com.

ing that the scale of the reseller is larger. By contrast, the largest e-commerce platform is organized as a marketplace in most developing countries, except India. In addition to the examples from the United States and China, cross-country evidence also suggests that marketplace dominance is common in developing countries, where firm size distribution tends to skew towards small firms.

To supplement the consensus that the firm size distribution skews more towards small firms in poor countries than in rich countries, we also compared comparable firm size distributions among four European countries in Table 4: Germany, the United Kingdom, Spain, and Italy. Compared to Germany and Britain, both Spain and Italy have relatively lower GDP per capita; thus, they also have a larger proportion of small firms, as shown in Appendix A.2. This also coincides with the skewing of firm size distribution towards small firms.

Although the above evidence is far from conclusive as we only considered the relative scale of the two or three largest rather than all platforms, it still indicates that the relative scale of the reseller to the marketplace is larger in developed countries than in developing countries, when multi-homing of each firm is permitted in reality—firms reside in small platforms also sell through the largest platform. In the following section, a tractable model was developed to shed light on the relationship between offline firm size distribution and platform scale, where offline firm size only affects the demand on platforms. We also considered the situation in which the firm size affects the marginal cost of production as well in Section 5.

# 4 Model

Our model will show how offline firm size distribution shapes the scale of the two platforms. The distinction between the two platforms is characterized by different pricing mechanisms. Platforms know only the firm size distribution but not each firm's size. Due to the enormous number of firms in the economy, every single offline firm is so minuscule relative to a platform that no bargain occurs between them. The reseller optimally sets centralized purchasing and selling prices for all firms. By contrast, the marketplace decentralizes pricing to individual firms, who set the selling prices depending on their demand. Although both platforms coexist, the competition between the reseller and the marketplace is not considered in our model. An offline firm decides to sell via a platform as long as it is profitable to do so. Thus the multi-homing of a firm allows us to focus on the largest two platforms, because of the pecking order in firms' multi-homing.

E-commerce platforms enable firms to access a large market. We model such a demand expansion in an ad-hoc way. We let $k$ denote the size of a firm, which may represent either the capital or labor input level. The demand of a firm with size $k$ through the platform is

$$q = q(p, k) = k^{\theta} p^{-\sigma},$$

where $p$ and $q$ are the price and quantity to the final consumers. $\sigma > 1$ denotes the constant demand elasticity and $\theta \geq 0$ characterizes the degree to which the demand expands with the firm size $k$. $\theta = 0$ implies that the demand is independent of the firm size. When $\theta > 0$, a large firm is associated with a high demand.

On the cost side, the marginal cost of each firm to produce the good is $c$, independent of the firm's size $k$. The specification assumes away the cost heterogeneity across firms, where larger firms producing the same products tend to have lower marginal costs. This assumption will be relaxed to a degree in the next section. Moreover, a fixed cost $f$ is incurred if a firm sells via a platform. To better characterize the pricing mechanism, the fixed cost $f$ that determines the entry of a firm is assumed to be the same for all firms in every platform. Such simplification of the cost factors allows us to focus on the demand side, through which the platform functions more significantly.

To characterize the offline firm size distribution, we assume the firm size $k$ follows a Pareto distribution (Axtell, 2001; Gabaix, 2009; Sutton, 1997) with a cumulative distribution function

$$G(k) = 1 - \frac{1}{k^{\alpha}}$$

and the probability density function $g(k) = \alpha k^{-\alpha-1}$, where $\alpha > 0$ is the key parameter that governs the firm size distribution.[6] Given there is a larger proportion of small offline firms in China than in the United States, we have $\alpha_{\mathrm{CN}} > \alpha_{\mathrm{US}}$ in our model because a large $\alpha$ implies that the distribution skews towards small firms. Additionally, the Pareto distribution gives the mean and the variance of the firm size

$$\mathbb{E}(k) = \frac{\alpha}{\alpha - 1}, \quad \mathrm{Var}(k) = \frac{\alpha}{(\alpha-1)^2(\alpha-2)},$$

provided that $\alpha > 2$. So $\alpha_{\mathrm{CN}} > \alpha_{\mathrm{US}}$ also means that the average firm size and its dispersion are larger in the United States than in China, which is consistent with the cross-country evidence on the firm size distribution (Poschke, 2018). Besides, the nice property of the Pareto distribution yields closed-form solutions in the following analysis.

Regarding the values of parameters $\theta$ and $\alpha$, we assume $0 < \theta < \alpha$. Because online markets empower firms to access a larger market, such an assumption on the bounds of $\theta$ allows the de-

[6]Here we assume the minimal size of a firm is $\underline{k} = 1$ for analytical simplicity.

mand to be positively correlated with the firm size, but may not be too responsive so that the total quantity of demand of the entire platform still keeps finite. The assumption of a relatively small $\theta$, especially $\theta < 1$, is also consistent with the common observation that the platform favors small firms by scaling down the demand discrepancy between the large and the small firms.

There are a few more practical concerns deserve attention. Firstly, because of product characteristics and asymmetric information online, the likelihood of a product being sold online may vary across product categories (Brynjolfsson and Smith, 2000; Lieber and Syverson, 2012). For example, standardized consumer products such as TV and consumer electronics are more likely to be sold online than personalized consumer products such as clothes. Correspondingly, our model posits that all firms are potential candidates to sell via platforms regardless of product categories. Secondly, the actual pricing mechanisms of the online platforms are quite flexible and hybrid in reality (Farronato, 2018). For example, eBay adopts both auctions and posted-price selling. In this regard, our model distinguishes the marketplace and the reseller by the most critical difference in their pricing mechanisms. Lastly, the payment structure between the firms and the online platforms is usually composed of several parts, mainly the commission and subscription fees (Weyl, 2010). While the commission fee affects the usage, the subscription fee limits the participation. As for large platform, we focus on the commission fee rather than the subscription fee.

Since the competition between the reseller and the marketplace is assumed away, we discuss the two platforms separately as follows.

### 4.1 Marketplace

A marketplace does not engage in selling by itself. To decentralize the price, the marketplace signs a revenue-sharing contract with each firm selling on it and charges a proportion $\lambda \in (0, 1)$ of its revenue. A firm accepts the contract if it is profitable.

**Firm:** Given the revenue-sharing proportion $\lambda$, a firm with size $k$ maximizes the profit $\pi_M$:

$$\max_p \pi_M(k) = (1-\lambda)pq - cq - f = [(1-\lambda)p - c]k^\theta p^{-\sigma} - f,$$

where $c$ is the size-independent marginal cost and $f$ is the fixed cost of operation on the marketplace. $(1-\lambda)pq$ is the actual revenue for the firm after deducting the share charged by the marketplace. The first order condition generates

$$p = \frac{\sigma}{\sigma - 1}\left(\frac{c}{1-\lambda}\right).$$

We can see the firm's optimal selling price is independent of its size. Given the optimal price, the quantity of demand and the profit of the firm are

$$q = k^{\theta} \left[ \frac{\sigma}{\sigma - 1} \left( \frac{c}{1 - \lambda} \right) \right]^{-\sigma}, \quad \pi_M(k) = k^{\theta} \left[ \frac{\sigma}{\sigma - 1} \left( \frac{c}{1 - \lambda} \right) \right]^{-\sigma} \frac{c}{\sigma - 1} - f.$$

If the profit of outside option is normalized as 0, the firm sells on the marketplace if $\pi_M(k) \geq 0$, or equivalently,

$$k \geq \left[ \frac{c^{\sigma-1}\sigma^{\sigma}}{(\sigma-1)^{\sigma-1}} f \right]^{\frac{1}{\theta}} \left( \frac{1}{1-\lambda} \right)^{\frac{\sigma}{\theta}} \equiv \underline{k}_M.$$

Because firm's profit increases with its size $k$, it means only those firms with size greater than the cutoff $\underline{k}_M$ will sell on the marketplace.[7]

**Marketplace:** Given the total number of firms $N$ in the country and the firm size distribution $G(k)$, the marketplace sets the optimal revenue-sharing proportion $\lambda$ to maximize its profit $\Pi_M$:

$$\begin{aligned} \max_{\lambda} \Pi_M &= N \int_{\underline{k}_M}^{\infty} \lambda p q(p,k) \, \mathrm{d}G(k) = N \int_{\underline{k}_M}^{\infty} \lambda \alpha \left[ \frac{\sigma}{\sigma - 1} \left( \frac{c}{1 - \lambda} \right) \right]^{1-\sigma} k^{\theta-\alpha-1} \, \mathrm{d}k \\ &= \frac{N\lambda\alpha}{\alpha - \theta} \left[ \frac{\sigma}{\sigma - 1} \left( \frac{c}{1 - \lambda} \right) \right]^{1-\sigma} (\underline{k}_M)^{\theta-\alpha}. \end{aligned}$$

The first order condition generates

$$\lambda = \frac{\theta}{\alpha\sigma}. \tag{1}$$

Under the assumption $\theta < \alpha$, we have $\lambda < \frac{1}{\sigma} < 1$ since $\sigma > 1$. Given the marketplace sets a revenue-sharing proportion according to (1), the size of the smallest firm selling on the marketplace is

$$\underline{k}_M = \left[ \frac{c^{\sigma-1}\sigma^{\sigma}}{(\sigma-1)^{\sigma-1}} f \right]^{\frac{1}{\theta}} \left( \frac{\alpha\sigma}{\alpha\sigma - \theta} \right)^{\frac{\sigma}{\theta}}. \tag{2}$$

### 4.2 Reseller

Unlike the marketplace on which each firm sets a price individually based on its size and thus demand, the reseller purchases products from the firms at a centralized wholesale price $s$ and resells them on its own. Similarly, a firm chooses to sell its product to the reseller if it is profitable. Let $\mathcal{K} \subseteq (1, +\infty)$ be the collection of firms that choose to sell to the reseller, then if the reseller sells the product at price $p$, the aggregate demand is

$$Q(p) = N \int_{\mathcal{K}} q(p,k) \, \mathrm{d}G(k),$$

[7] To be more realistic, we assume that the marginal cost of producing the product, $c$, or the fixed cost of selling through the online platforms, $f$, is high enough so that it will not be profitable for some of the smallest firms to sell online. More specifically, we assume that $\sigma c^{\sigma-1} f \geq 1$ so that $\underline{k}_M > 1$ in equilibrium. See Appendix A.3 for detailed derivation.

and the reseller's profit is

$$\Pi_R = (p-s)Q(p) = (p-s)p^{-\sigma} N \int_{\mathcal{K}} k^{\theta} \, \mathrm{d}G(k).$$

Note that only the wholesale price $s$ affects the firm's decision about whether to sell to the reseller, so at a fixed cost $s$, the set $\mathcal{K}$ is also fixed. Given $s$, the reseller's problem is to set $p$ to maximize the profit, which gives the optimal selling price:

$$p = \frac{\sigma}{\sigma-1} s.$$

**Firm:** When the reseller offers a wholesale price $s$, the demand of a firm with size $k$ is

$$q = k^{\theta} p^{-\sigma} = k^{\theta} \left( \frac{\sigma s}{\sigma - 1} \right)^{-\sigma}.$$

A small firm sells through the reseller when its profit outweighs zero–the outside option; that is,

$$\pi_R(k) = (s-c)q - f = (s-c)k^{\theta} \left( \frac{\sigma s}{\sigma-1} \right)^{-\sigma} \geq 0 \implies k \geq \left[ \frac{1}{s-c} \left( \frac{\sigma s}{\sigma-1} \right)^{\sigma} f \right]^{\frac{1}{\theta}} \equiv \underline{k}_R.$$

Just like the marketplace case, only those firms with size greater than the cutoff $\underline{k}_R$ will choose to sell to the reseller.[8]

**Reseller:** Given the selling price $p = \frac{\sigma}{\sigma-1} s$, the reseller sets the optimal wholesale price $s$ to maximize the profit $\Pi_R$:

$$\begin{aligned} \max_s \Pi_R &= N \int_{\underline{k}_R}^{\infty} (p-s) q(p,k) \, \mathrm{d}G(k) = N \int_{\underline{k}_R}^{\infty} \frac{s\alpha}{\sigma-1} \left( \frac{\sigma s}{\sigma-1} \right)^{-\sigma} k^{\theta-\alpha-1} \, \mathrm{d}k \\ &= \frac{N s \alpha}{(\alpha-\theta)(\sigma-1)} \left( \frac{\sigma s}{\sigma-1} \right)^{-\sigma} (\underline{k}_R)^{\theta-\alpha}. \end{aligned}$$

The first order condition generates

$$s = \frac{\alpha\sigma - \theta}{\alpha(\sigma-1)} c. \tag{3}$$

When the reseller sets the wholesale price $s$ according to (3), the minimal firm size at which the profit starts exceeding 0 is

$$\underline{k}_R = \left[ \frac{c^{\sigma-1}\sigma^{\sigma}}{(\sigma-1)^{\sigma-1}} f \right]^{\frac{1}{\theta}} \left[ \frac{\alpha\sigma-\theta}{\alpha(\sigma-1)} \right]^{\frac{\sigma}{\theta}} \left( \frac{\alpha}{\alpha-\theta} \right)^{\frac{1}{\theta}}. \tag{4}$$

[8] Similarly, we assume here that $\sigma c^{\sigma-1} f \geq 1$ to make $\underline{k}_R > 1$. See Appendix A.3 for detailed derivation.

In the following part, we will discuss how the offline firm size distribution affects the firm size cutoffs separating the offline-only firms and the firms selling online, the marketplace's and the reseller's strategies, and the relative scale of the two platforms.

### 4.3 Firm Size Distribution and Platform Scale

We aim to examine the relationship between the scales of the two platforms and the offline firm size distribution. In the evidence presented in previous section, the scale of each platform is measured by the GMV, which corresponds to the total revenue in our model, i.e.

$$S_j = N \int_{\underline{k}_j}^{\infty} p_j q(p_j, k)\, \mathrm{d}G(k), \quad j \in \{M, R\}.$$

Note that for both the marketplace and the reseller, the selling prices are independent of the firm sizes, so for platform $j \in \{M, R\}$,

$$S_j = N \int_{\underline{k}_j}^{\infty} p_j \cdot p_j^{-\sigma} k^{\theta}\, \mathrm{d}G(k) = \frac{N}{p_j^{\sigma-1}} \cdot \frac{\alpha}{\alpha-\theta} (\underline{k}_j)^{\theta-\alpha}.$$

We can see that the scale of the platform is determined by three factors: the entire market size represented by the total number of firms $N$ in the economy, the selling price $p_j$ that is connected to the revenue of each firm, and the platform-augmented demand component $\frac{\alpha}{\alpha-\theta}(\underline{k}_j)^{\theta-\alpha}$. The platform-augmented demand component is only attributed to the proportion of the offline firms selling online, which depends on the cutoff $\underline{k}_j$ and the shape of firm size distribution governed by $\alpha$.

Because the market size $N$ is exogenously given, we will derive comparative statics regarding the latter two factors—the selling prices and the platform-augmented demand component, making them the cornerstones of the relative scale.

**Proposition 1.** *Under the assumption $0 < \theta < \alpha$ and $\sigma > 1$, the selling price via the marketplace $p_M$ decreases in $\alpha$ and the selling price of the reseller $p_R$ increases in $\alpha$, that is,*

$$\frac{\partial p_M}{\partial \alpha} < 0, \quad \frac{\partial p_R}{\partial \alpha} > 0.$$

*Proof.* From (1) and (3), the optimal revenue-sharing proportion $\lambda$ for the marketplace and the optimal wholesale price $s$ for the reseller are

$$\lambda = \frac{\theta}{\alpha\sigma}, \quad s = \frac{\alpha\sigma-\theta}{\alpha(\sigma-1)} c.$$

Then, we have

$$\frac{\partial \lambda}{\partial \alpha} = -\frac{\theta}{\alpha^2 \sigma} < 0, \quad \frac{\partial s}{\partial \alpha} = \frac{\theta}{\alpha^2(\sigma - 1)} > 0.$$

Both the proportion $(1 - \lambda)$ of the selling price that a firm gets from the marketplace and the wholesale price $s$ that a firm receives from the reseller are increasing in $\alpha$. As a result, when the distribution of firm size is more skewed towards small firms, the selling price on the marketplace $p_M = \frac{\sigma}{\sigma-1} \cdot \frac{c}{1-\lambda}$ becomes cheaper while the selling price of the reseller $p_R = \frac{\sigma s}{\sigma-1}$ becomes more expensive. □

According to Proposition 1, as the firm size distribution is skewed towards small firms, $p_R/p_M$ increases as these two final prices go in opposite directions. We can further show that when $\alpha$ is sufficiently large, the selling price on the marketplace will become cheaper than the price via the reseller.

**Lemma 1.** *Given $\sigma > 1$, the marketplace has cheaper selling price than the reseller, i.e. $p_M < p_R$, if and only if*

$$\alpha > \frac{\theta}{\sigma - \sqrt{\sigma(\sigma - 1)}}. \tag{5}$$

*Proof.* The selling prices of both the marketplace and the reseller are

$$p_M = \frac{\alpha\sigma^2}{(\sigma - 1)(\alpha\sigma - \theta)} c, \quad p_R = \frac{(\alpha\sigma - \theta)\sigma}{\alpha(\sigma - 1)^2} c.$$

Thus we have

$$\frac{p_M}{p_R} = \sigma(\sigma - 1)\left(\frac{\alpha}{\alpha\sigma - \theta}\right)^2 = \left[\frac{\sqrt{\sigma(\sigma - 1)}}{\sigma - \frac{\theta}{\alpha}}\right]^2.$$

Therefore, $p_M < p_R$ if and only if $\sigma - \frac{\theta}{\alpha} > \sqrt{\sigma(\sigma - 1)}$ or equivalently $\alpha > \frac{\theta}{\sigma - \sqrt{\sigma(\sigma-1)}}$. □

Since $0 < \sigma - \sqrt{\sigma(\sigma - 1)} < 1$ when $\sigma > 1$, condition (5) also implies $\theta < \alpha$ that we assumed earlier.

The following two propositions investigate the effect of change in the firm size distribution on the cutoffs and further the platform-augmented demand component.

**Proposition 2.** *Under the assumption $0 < \theta < \alpha$ and $\sigma > 1$, both the firm size cutoffs $\underline{k}_M$ $\underline{k}_R$ are decreasing in $\alpha$, that is,*

$$\frac{\partial \underline{k}_M}{\partial \alpha} < 0, \quad \frac{\partial \underline{k}_R}{\partial \alpha} < 0.$$

*Proof.* From (2) and (4), we can rewrite $\underline{k}_M$ and $\underline{k}_R$ as

$$\underline{k}_M = \left[\frac{c^{\sigma-1}\sigma^{2\sigma} f}{(\sigma-1)^{\sigma-1}}\right]^{\frac{1}{\theta}} \left(\frac{\alpha}{\alpha\sigma-\theta}\right)^{\frac{\sigma}{\theta}}, \quad \underline{k}_R = \left[\frac{c^{\sigma-1}\sigma^{\sigma} f}{(\sigma-1)^{2\sigma-1}}\right]^{\frac{1}{\theta}} \left(\frac{\alpha\sigma-\theta}{\alpha}\right)^{\frac{\sigma}{\theta}} \left(\frac{\alpha}{\alpha-\theta}\right)^{\frac{1}{\theta}},$$

which lead to

$$\frac{\partial \underline{k}_M}{\partial \alpha} \propto \frac{\partial}{\partial \alpha}\left[\left(\frac{\alpha}{\alpha\sigma-\theta}\right)^{\frac{\sigma}{\theta}}\right] = -\left(\frac{\alpha}{\alpha\sigma-\theta}\right)^{\frac{\sigma}{\theta}-1} \frac{\sigma}{(\alpha\sigma-\theta)^2} < 0,$$

$$\frac{\partial \underline{k}_R}{\partial \alpha} \propto \frac{\partial}{\partial \alpha}\left[\left(\frac{\alpha\sigma-\theta}{\alpha}\right)^{\frac{\sigma}{\theta}} \left(\frac{\alpha}{\alpha-\theta}\right)^{\frac{1}{\theta}}\right] = -\left(\frac{\alpha\sigma-\theta}{\alpha}\right)^{\frac{\sigma}{\theta}-1} \left(\frac{\alpha}{\alpha-\theta}\right)^{\frac{1}{\theta}-1} \frac{\theta(\sigma-1)}{\alpha(\alpha-\theta)^2} < 0. \qquad \square$$

Therefore, as the offline firm size distribution is skewed towards small firms, corresponding to a larger $\alpha$, the firm size cutoffs $\underline{k}_R$ and $\underline{k}_M$ both decrease. This suggests that in a economy with relatively more small or medium-sized firms, small firms are more likely to sell online through the marketplace or reseller. In addition, the above cutoffs also suggest

$$\frac{\underline{k}_M}{\underline{k}_R} = \left(\frac{p_M}{p_R}\right)^{\frac{\sigma}{\theta}} \left(\frac{\alpha-\theta}{\alpha}\right)^{\frac{1}{\theta}}.$$

Because $0 < \theta < \alpha$, $\underline{k}_M < \underline{k}_R$ whenever $p_M < p_R$. This indicates the marketplace is more favorable to small firms than the reseller, since the firm size cutoff for the marketplace is smaller than the cutoff for the reseller.

Based on this result, the next proposition shows how the platform-augmented demand component

$$w_j = \int_{\underline{k}_j}^{\infty} k^{\theta}\, \mathrm{d}G(k) = \frac{\alpha}{\alpha-\theta}(\underline{k}_j)^{\theta-\alpha}, \quad j \in \{M, R\},$$

will change in response to $\alpha$.

**Proposition 3.** *Under the assumption* (5) *and* $\sigma > 1$,

$$\frac{\partial \ln w_M}{\partial \alpha} > \frac{\partial \ln w_R}{\partial \alpha} \quad \textit{or equivalently} \quad \frac{\partial (w_M/w_R)}{\partial \alpha} > 0.$$

*Proof.* Since

$$\frac{w_M}{w_R} = \left(\frac{\underline{k}_M}{\underline{k}_R}\right)^{\theta-\alpha},$$

we have

$$\frac{\partial \ln(w_M/w_R)}{\partial \alpha} = \frac{\partial \ln w_M}{\partial \alpha} - \frac{\partial \ln w_R}{\partial \alpha}$$

$$= \left[\frac{\theta}{\alpha(\theta-\alpha)} + \sigma(\alpha-\theta) - \ln \underline{k}_M\right] - \left[\frac{\theta}{\alpha(\theta-\alpha)} + \theta(\sigma-1) - \ln \underline{k}_R\right]$$
$$= \alpha(2\sigma-1)\left(\frac{\sigma}{2\sigma-1} - \frac{\theta}{\alpha}\right) - \ln \frac{\underline{k}_M}{\underline{k}_R}.$$

Because for $\sigma > 1$,

$$\sigma - \sqrt{\sigma(\sigma-1)} = \frac{\sigma^2 - \sigma(\sigma-1)}{\sigma + \sqrt{\sigma(\sigma-1)}} < \frac{\sigma}{\sigma + \sqrt{(\sigma-1)^2}} = \frac{\sigma}{2\sigma-1},$$

(5) implies

$$\frac{\theta}{\alpha} < \sigma - \sqrt{\sigma(\sigma-1)} < \frac{\sigma}{2\sigma-1} \implies \frac{\sigma}{2\sigma-1} - \frac{\theta}{\alpha} > 0$$

and by Lemma 1,

$$p_M < p_R \implies \underline{k}_M < \underline{k}_R \implies \ln \frac{\underline{k}_M}{\underline{k}_R} < 0,$$

it follows that

$$\frac{\partial \ln(w_M/w_R)}{\partial \alpha} > 0 \implies \frac{\partial (w_M/w_R)}{\partial \alpha} > 0. \qquad \square$$

Proposition 3 shows that the demand component attributed to the proportion of firms selling on the marketplace, $w_M$, is more responsive to $\alpha$ than that via the reseller, $w_R$. Although smaller and smaller firms participate in selling on the online platforms as the firm size cutoffs decrease in $\alpha$, implied by Proposition 2, the proportion of firms selling via the platform $j \in \{M, R\}$, $\int_{\underline{k}_j}^{\infty} \mathrm{d}G(k)$, and the associated platform-augmented demand component $w_j$, do not necessarily increase, because they rely on both the firm size cutoff $\underline{k}_j$ and the shape of firm size distribution governed by $\alpha$. In particular, when $\underline{k}_j$ is large, the proportion of firms and the platform-augmented demand component could be decreasing in $\alpha$; by contrast, they will increase in $\alpha$ when $\underline{k}_j$ is small (see Appendix A.4).

In the above analyses, we have investigated the effect of $\alpha$ on the prices, the firm size cutoffs, and the platform-augmented demand components. Finally, we put these results together and take the model to the evidence before—the relative scale of the reseller to the marketplace. Because the scale of each platform is measured by the GMV, the relative scale of the reseller to the marketplace is formulated as

$$r = \frac{S_R}{S_M},$$

where $S_R$ and $S_M$ denote the respective total revenue of the reseller and the marketplace. The following is our main proposition.

**Proposition 4.** *Under the assumption* (5) *and* $\sigma > 1$*, the relative scale* $r$ *is decreasing in* $\alpha$*, that is,*

$$\frac{\partial r}{\partial \alpha} < 0.$$

*Proof.* See Appendix A.5. □

The above proposition illustrates that the relative scale of the reseller to the marketplace decreases as the offline firm size distribution is skewed towards small firms. Because relative to the United States, the firm size distribution in China is more skewed towards small firms, i.e., $\alpha_{\mathrm{CN}} > \alpha_{\mathrm{US}}$, the proposition explains the difference in the relative scale of the reseller to the marketplace in the United States and China as shown in Table 1. Also, because the developing countries tend to have a larger value of $\alpha$ in general and share similar firm size distribution to China, this proposition also provides an explanation for the cross-country pattern demonstrated in Figure 1.

The intuition behind Proposition 4 is as follows. As the scales of a platform $j \in \{M, R\}$ can be written as

$$S_j = \frac{N w_j}{p_j^{\sigma-1}},$$

a platform faces the trade-off of extracting more profit from each firm at the expense of attracting less firms. When the offline firm size distribution is skewing towards small firms, i.e., $\alpha$ increases, $p_R/p_M$ increases as these two final prices go in opposite directions, implied by Proposition 1. Thus, the revenue contributed by each firm to the reseller relative to the marketplace increases, making the marketplace more attractive to the consumers. On the other hand, according to Proposition 3, the platform-augmented demand component for the marketplace, $w_M$, due to potentially higher proportion of firms selling on the marketplace, increases more than that of the reseller, $w_R$. In consequence, the marketplace attains a larger relative scale to the reseller.

One critical condition for the proposition is that the selling price on the marketplace need to be lower than the selling price set by the reseller: $p_M < p_R$, or in terms of the model primitives, condition (5). This condition is often satisfied in reality. To validate it, we manually collected the prices of different products from the two most prominent platforms in the United States (Amazon and eBay) and the two largest in China (JD and Alibaba). To rule out the concern of product differentiation, we collect the information of the same products sold on both platforms in each country (that is, sold through both the marketplace and the reseller in the United States or in China) and include products are in different categories such as electronics, apparatus, and book shelves. An example of identical products sold on Amazon and eBay are shown in Appendix A.6. To avoid dynamic pricing issue, the price of each product is tracked consecutively for three days, from January 24 to January 26, 2022. We also drop those products with huge heterogeneous sales volumes between the two platforms to ensure the posted prices do reflect the selling price. Then we run the following regression in each country:

$$p_{it} = \beta D_{\text{Reseller}} + \mu_i + \delta_t + \epsilon_{it},$$

where $i$ and $t$ index the product and time, and $p_{it}$ denotes the posted price of product $i$ at time $t$. The dummy variable $D_{\text{Reseller}} = 1$ if the product is sold via a reseller. Given the prices of the same products are collected from two platforms, the product fixed effect $\mu_i$ absorbs the product-specific information and the time fixed effect $\delta_t$ is used to control systematic trend in price variation. The estimation results are in Table 2.

Table 2: Validate $p_M < p_R$ in the United States and China

| Selling price | United States (USD) | China (CNY) |
|---|---|---|
| Reseller dummy $\beta$ | $93.99^{**}$ | $16.72^{**}$ |
| | (29.81) | (7.09) |
| Obs | 888 | 1728 |
| $R^2$ | 0.98 | 0.99 |

Standard errors are in parentheses. $**$ denotes significance at the 5% level.

Because different products are sold in the United States and China, the price gap is not comparable. However, the estimates confirm that the price of the same product sold by the reseller is indeed higher than that on the counterpart marketplace in both countries.

The model's implication is consistent with the empirical fact about the relative scale of the reseller to the marketplace, but due to the constant marginal cost assumption, the model also implies a uniform selling prices on the marketplace across all firm with different sizes, which is far from reality. In the following section, we investigated an extended model with a relaxed assumption.

## 5 Extension

In previous model, the size-independent marginal cost generates a uniform selling prices on both platforms. That is, the selling prices on the marketplace $p_M = \frac{\alpha\sigma^2}{(\alpha\sigma-\theta)(\sigma-1)}c$, like the selling price $p_R$ of the reseller, are the same for all firms regardless of sizes. However, larger firms in reality tend to have lower marginal costs. In this section, previous setup is extended to adapt to this observation and allows for size-dependent marginal cost of production.

Given the firm size now affects both the demand and the cost, we assumed the marginal cost of a firm of size $k$ is

$$c = c(k) = c_0 k^{-\phi}, \quad \text{where } \phi > 0.$$

Meanwhile, the demand for firm of size $k$ on a platform is still $q = q(p, k) = k^{\theta} p^{-\sigma}$. So a large firm has both a high demand and a low marginal cost. Obviously, $\phi = 0$ in previous constant marginal cost case. For the sake of tractability, we further assumed $\phi = \theta$. Under condition $\sigma > 1$ and $\alpha > \theta\sigma > 0$, similar derivations give the three pairs of key variables as follows:[9] the selling prices via the marketplace and the reseller

$$p_M(k) = \frac{\alpha\sigma}{(\alpha - \theta)(\sigma - 1)} c_0 k^{-\theta}, \quad p_R = \left( \frac{\alpha - \theta\sigma}{\alpha\sigma - \alpha} \cdot \frac{c_0}{f} \right)^{\frac{1}{\sigma}}; \tag{6}$$

the respective firm size cutoffs

$$\underline{k}_M = \left( \frac{\alpha}{\alpha - \theta} \right)^{\frac{1}{\theta}} \left[ \frac{\sigma^{\sigma} c_0^{\sigma - 1} f}{(\sigma - 1)^{\sigma - 1}} \right]^{\frac{1}{\theta\sigma}}, \quad \underline{k}_R = \left[ \frac{(\alpha - \theta)(\alpha - \theta\sigma)^{-\frac{1}{\sigma}}}{\alpha^{1 - \frac{1}{\sigma}}} \right]^{\frac{1}{\theta}} \left[ \frac{\sigma^{2\sigma} c_0^{\sigma - 1} f}{(\sigma - 1)^{2\sigma - 1}} \right]^{\frac{1}{\theta\sigma}}; \tag{7}$$

and the respective scales of the marketplace and the reseller

$$S_M = N \int_{\underline{k}_M}^{\infty} p_M(k) q(p_M(k), k) \, \mathrm{d}G(k) = N \frac{\alpha^{2 - \frac{\alpha}{\theta}} \sigma^{1 - \frac{\alpha}{\theta}} c_0^{\frac{\alpha}{\theta\sigma} - \frac{\alpha}{\theta}} f^{1 - \frac{\alpha}{\theta\sigma}}}{(\alpha - \theta)^{1 - \frac{\alpha}{\theta}} (\alpha - \theta\sigma)(\sigma - 1)^{\frac{\alpha}{\theta\sigma} - \frac{\alpha}{\theta}}},$$
$$S_R = N \int_{\underline{k}_R}^{\infty} p_R q(p_R, k) \, \mathrm{d}G(k) = N \frac{\alpha^{1 + \frac{\alpha}{\theta} - \frac{\alpha}{\theta\sigma}} \sigma^{2 - 2\frac{\alpha}{\theta}} c_0^{\frac{\alpha}{\theta\sigma} - \frac{\alpha}{\theta}} f^{1 - \frac{\alpha}{\theta\sigma}}}{(\alpha - \theta)^{\frac{\alpha}{\theta}} (\alpha - \theta\sigma)^{1 - \frac{\alpha}{\theta\sigma}} (\sigma - 1)^{\frac{\alpha}{\theta\sigma} - \frac{2\alpha}{\theta} + 1}}.$$

It is worth noting that since the marginal cost diminishes as the size of the firm grows, now the selling price will vary between firms of different sizes when selling through the marketplace, but the selling price through the reseller still keeps constant across firms due to the centralized pricing scheme.

Based on the analyses, the following proposition characterizes the comparative statics of prices and firm size cutoffs in response to $\alpha$, which maintain the same implications in Proposition 1 and Proposition 2.

**Proposition 5.** *Under the assumption $\sigma > 1$ and $\alpha > \theta\sigma > 0$:*

*(i) The selling price via the marketplace is decreasing in $\alpha$ whereas the selling price via the reseller is decreasing in $\alpha$; that is,*

$$\frac{\partial p_M(k)}{\partial \alpha} < 0, \quad \frac{\partial p_R}{\partial \alpha} > 0.$$

[9] The detailed derivations are collected in Appendix A.7. Here we also assume that $\sigma c_0^{\sigma-1} f \geq 1$ such that the cutoffs $\underline{k}_M > 1$ and $\underline{k}_R > 1$ (see Appendix A.3).

*(ii) Both firm size cutoffs $\underline{k}_M$ and $\underline{k}_R$ are decreasing in $\alpha$, that is,*

$$\frac{\partial \underline{k}_M}{\partial \alpha} < 0, \quad \frac{\partial \underline{k}_R}{\partial \alpha} < 0.$$

*Proof.* Because $\sigma > 1 \implies \alpha > \theta\sigma > \theta > 0$, by (6), the effect of $\alpha$ on the selling prices are

$$\frac{\partial p_M(k)}{\partial \alpha} = \frac{\sigma c_0 k^{-\theta}}{\sigma - 1} \cdot \frac{\partial}{\partial \alpha}\left(\frac{\alpha}{\alpha - \theta}\right) = -\frac{\sigma c_0 k^{-\theta}}{\sigma - 1} \cdot \frac{\theta}{(\alpha - \theta)^2} < 0;$$

$$\frac{\partial p_R}{\partial \alpha} = \frac{c_0}{\sigma f}\left(\frac{\alpha - \theta\sigma}{\alpha\sigma - \alpha} \cdot \frac{c_0}{f}\right)^{\frac{1}{\sigma}-1} \frac{\partial}{\partial \alpha}\left(\frac{\alpha - \theta\sigma}{\alpha\sigma - \alpha}\right) = \frac{c_0}{\sigma f}\left(\frac{\alpha - \theta\sigma}{\alpha\sigma - \alpha} \cdot \frac{c_0}{f}\right)^{\frac{1}{\sigma}-1} \frac{\theta\sigma}{\alpha^2(\sigma - 1)} > 0.$$

For the effect of $\alpha$ on the firm size cutoffs, by (7),

$$\frac{\partial \underline{k}_M}{\partial \alpha} = -\left[\frac{\sigma^\sigma c_0^{\sigma-1} f}{(\sigma - 1)^{\sigma-1}}\right]^{\frac{1}{\theta\sigma}} \frac{\alpha^{\frac{1}{\theta}-1}}{(\alpha - \theta)^{\frac{1}{\theta}+1}} < 0,$$

$$\frac{\partial \underline{k}_R}{\partial \alpha} = -\left[\frac{\sigma^2 c_0^{1-\frac{1}{\sigma}} f^{\frac{1}{\sigma}}}{(\sigma - 1)^{2-\frac{1}{\sigma}}}\right]^{\frac{1}{\theta}} \cdot \frac{1}{\theta}\left[\frac{(\alpha - \theta)(\alpha - \theta\sigma)^{-\frac{1}{\sigma}}}{\alpha^{1-\frac{1}{\sigma}}}\right]^{\frac{1}{\theta}-1} \frac{\theta^2(\sigma - 1)(\alpha - \theta\sigma)^{-\frac{1}{\sigma}-1}}{\alpha^{2-\frac{1}{\sigma}}} < 0. \qquad \square$$

Given the selling price now can very with the firm size, it is no longer straightforward to define the platform-augmented demand component as before. But as shown in the following proposition, the relative scale of the reseller to the marketplace still demonstrates similar comparative statics with respect to $\alpha$.

**Proposition 6.** *Under the assumption $\alpha > \frac{\sigma\theta}{\sigma - \sqrt{\sigma(\sigma-1)}}$ and $\sigma > 1$, the relative scale $r = S_R/S_M$ is decreasing in $\alpha$, that is,*

$$\frac{\partial r}{\partial \alpha} < 0.$$

*Proof.* See Appendix A.8. $\square$

Proposition 6 suggests the relative scale of the reseller to the marketplace decreases as the offline firm size distribution is skewed towards small firms, showing the same result as Proposition 4. Interestingly, the condition $\alpha > \frac{\theta\sigma}{\sigma - \sqrt{\sigma(\sigma-1)}}$ in Proposition 6 is very similar to the condition $\alpha > \frac{\theta}{\sigma - \sqrt{\sigma(\sigma-1)}}$ in Proposition 4. This is because when the marginal cost becomes size-dependent $c = c_0 k^{-\theta}$, both the demand and the marginal cost depend on the firm size: while the demand is expanded by $k^\theta$, the cost is reduced by $k^{-\theta}$. Consequently, the effect of firm size on the platform's total revenue is expanded proportionally to $k^\theta \cdot k^{-\theta(1-\sigma)} = k^{\theta\sigma}$, where $\theta\sigma$ is isomorphic to $\theta$ in the model with size-independent marginal cost.

In a nutshell, when the marginal cost is size-dependent, the selling price via marketplace are heterogeneous; nevertheless, the implications regarding the prices, the firm size cutoffs and the relative scale are all preserved.

# 6 Conclusion

Platforms in the online market develop at a fast pace, but cannot grow without offline firms. Motivated by the phenomenon that the relative scale of the reseller to the marketplace is larger in the United States than in China, this study first extended the motivation beyond the United States and China, showing that the pattern of the relative scale of the reseller to the marketplace is common in developed and developing countries. To account for this phenomenon, this study proposed offline firm size distribution, one of the most important fundamentals that varies with development level, as the determinant of platform scale. We developed a tractable model that links platform scale with offline firm size distribution through different pricing mechanisms. While the reseller is characterized by establishing a common wholesale price, the marketplace decentralizes pricing for individual firms by proportionally charging revenue. Consequently, the marketplace favors a larger base of firms than the reseller, which tends to capture more profit from each firm. As offline firm size distribution is skewed towards small firms, the marketplace that attracts a larger proportion of firms tends to become larger. Therefore, the relative scale of the reseller to the marketplace is smaller, as evidenced by China and the United States. But admittedly, the Pareto distribution of the firm size generates closed-form solutions that facilitate us to explore the underlying mechanisms by decomposing the overall effect into several margins.

While studies have investigated the effect from online to offline, our study examined the effect from offline to online. Our study contributes to the literature on firm size distribution, a long-lasting topic in economic development, by showing that the connection to firm size distribution goes from offline to online.

However, the absence of a platform network effect in our study leaves much to be desired. Our study does not consider the indirect network effect in which demand depends on the number of firms. Given that the direct network effect intensifies competition among firms, the reseller is likely to internalize such an effect compared with the marketplace. However, this effect was not covered in our study, either. Furthermore, additional research should examine the hybrid mode adopted by Amazon and JD to address competition within a platform between first-party products and third-party products, given that the third-party share is not trivial.

# A Appendices

## A.1 The Scale of Platforms

Table 3: The Scale of Platforms: United States and China (2014/2015)

| | United States | | China | |
|---|---|---|---|---|
| | Reseller (Amazon) | Marketplace (eBay) | Reseller (JD) | Marketplace (Alibaba) |
| Third-party share in 2014 | 49% | 100% | 40% | 100% |
| GMV in 2014 (billion USD) | 166 | 82.8 | 38.3 | 239.7 |
| $r$ | 2.00 | | 0.16 | |
| $r^*$ | 0.52 | | 0.09 | |
| Third-party share in 2015 | 51% | 100% | 43% | 100% |
| GMV in 2015 (billion USD) | 225.6 | 81.3 | 63.8 | 349.1 |
| $r$ | 2.77 | | 0.18 | |
| $r^*$ | 0.35 | | 0.10 | |

Statistics are mainly from https://www.marketplacepulse.com and https://www.statista.com.

Table 4: Largest Platforms (2019)

| | No. 1 | No. 2 | No. 3 |
|---|---|---|---|
| *Developed Countries* | | | |
| United States | Amazon (R) | eBay (M) | |
| Canada | Amazon CA (R) | eBay CA (M) | |
| Japan | Amazon JP (R) | Rakuten (R) | Yahoo! JP (M) |
| Germany | Amazon DE (R) | eBay DE (M) | |
| United Kingdom | Amazon UK (R) | eBay UK (M) | |
| Spain | Amazon ES (R) | EI Corte Ingles (R) | eBay ES (M) |
| Italy | Amazon IT (R) | eBay IT (M) | |
| *Developing Countries* | | | |
| China | Alibaba (M) | JD (R) | |
| India | Amazon IN (R) | Flipkart (M) | |
| Brazil | Mercado Livre BR (M) | Americanas (R) | |
| Mexico | Mercado Livre MX (M) | Amazon MX (R) | |

(R) and (M) denote the reseller and the marketplace, respectively, though hybrid forms are prevalent.

## A.2 Firm Size Distribution

Table 5: Firm Size Distribution in European Countries (2006)

| | Germany | United Kingdom | Spain | Italy |
|---|---|---|---|---|
| 1–19 employees | 0.08 | 0.03 | 0.30 | 0.17 |
| 20–249 employees | 0.21 | 0.25 | 0.39 | 0.44 |
| $>250$ employees | 0.72 | 0.72 | 0.30 | 0.39 |
| GDP per capita (USD) | 37020 | 44095 | 28531 | 33341 |

Data is from Kalemli-Ozcan et al. (2015). The top part reports the share of employment accounted for by each corresponding size bin.

## A.3 Conditions for Cutoffs $\underline{k}_M > 1$ and $\underline{k}_R > 1$

In the constant marginal cost case, the firm size cutoffs $\underline{k}_M$ and $\underline{k}_R$ in equilibrium are

$$\begin{aligned}
\underline{k}_M &= \left[\frac{c^{\sigma-1}\sigma^{2\sigma}f}{(\sigma-1)^{\sigma-1}}\right]^{\frac{1}{\theta}}\left(\frac{\alpha}{\alpha\sigma-\theta}\right)^{\frac{\sigma}{\theta}} \\
&= \left(\sigma c^{\sigma-1}f\right)^{\frac{1}{\theta}}\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma-1}{\theta}}\left(\frac{\alpha\sigma}{\alpha\sigma-\theta}\right)^{\frac{\sigma}{\theta}}, \\
\underline{k}_R &= \left[\frac{c^{\sigma-1}\sigma^{\sigma}f}{(\sigma-1)^{2\sigma-1}}\right]^{\frac{1}{\theta}}\left(\frac{\alpha\sigma-\theta}{\alpha}\right)^{\frac{\sigma}{\theta}}\left(\frac{\alpha}{\alpha-\theta}\right)^{\frac{1}{\theta}} \\
&= \left(\sigma c^{\sigma-1}f\right)^{\frac{1}{\theta}}\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma-1}{\theta}}\left(\frac{\alpha\sigma-\theta}{\alpha\sigma-\alpha}\right)^{\frac{\sigma}{\theta}}\left(\frac{\alpha}{\alpha-\theta}\right)^{\frac{1}{\theta}}.
\end{aligned}$$

Because $\sigma > 1$ and $0 < \theta < \alpha$,

$$\frac{\sigma}{\sigma-1} > 1, \quad \frac{\alpha\sigma}{\alpha\sigma-\theta} > 1, \quad \frac{\alpha\sigma-\theta}{\alpha\sigma-\alpha} > 1, \quad \frac{\alpha}{\alpha-\theta} > 1.$$

In addition, the exponents $\frac{\sigma-1}{\theta} > 0$, $\frac{\sigma}{\theta} > 0$ and $\frac{1}{\theta} > 0$. So we have

$$\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma-1}{\theta}}\left(\frac{\alpha\sigma}{\alpha\sigma-\theta}\right)^{\frac{\sigma}{\theta}} > 1 \quad \text{and} \quad \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma-1}{\theta}}\left(\frac{\alpha\sigma-\theta}{\alpha\sigma-\alpha}\right)^{\frac{\sigma}{\theta}}\left(\frac{\alpha}{\alpha-\theta}\right)^{\frac{1}{\theta}} > 1.$$

Therefore, if $\sigma c^{\sigma-1}f \geq 1$ (or stronger, $cf \geq 1$), then $\underline{k}_M > 1$ and $\underline{k}_R > 1$.

In the extended model, when the marginal cost is size-dependent, i.e. $c = c_0 k^{-\theta}$, the cutoffs $\underline{k}_M$ and $\underline{k}_R$ are given by

$$\underline{k}_M = \left[\frac{\sigma^{\sigma}c_0^{\sigma-1}f}{(\sigma-1)^{\sigma-1}}\right]^{\frac{1}{\theta\sigma}}\left(\frac{\alpha}{\alpha-\theta}\right)^{\frac{1}{\theta}}$$

$$
\begin{aligned}
&= \left(\sigma c_0^{\sigma-1} f\right)^{\frac{1}{\theta\sigma}} \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma-1}{\theta\sigma}} \left(\frac{\alpha}{\alpha-\theta}\right)^{\frac{1}{\theta}}, \\
\underline{k}_R &= \left[\frac{\sigma^{2\sigma} c_0^{\sigma-1} f}{(\sigma-1)^{2\sigma-1}}\right]^{\frac{1}{\theta\sigma}} \left[\frac{(\alpha-\theta)(\alpha-\theta\sigma)^{-\frac{1}{\sigma}}}{\alpha^{1-\frac{1}{\sigma}}}\right]^{\frac{1}{\theta}} \\
&= \left(\sigma c_0^{\sigma-1} f\right)^{\frac{1}{\theta\sigma}} \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma-1}{\theta\sigma}} \left(\frac{\alpha\sigma-\theta\sigma}{\alpha\sigma-\alpha}\right)^{\frac{1}{\theta}} \left(\frac{\alpha}{\alpha-\theta\sigma}\right)^{\frac{1}{\theta\sigma}}.
\end{aligned}
$$

Because $\sigma > 1$ and $0 < \theta < \theta\sigma < \alpha$, we further have

$$
\frac{\alpha\sigma-\theta\sigma}{\alpha\sigma-\alpha} > 1, \quad \frac{\alpha}{\alpha-\theta\sigma} > 1.
$$

So given $\frac{\sigma-1}{\theta\sigma} > 0$, $\frac{1}{\theta\sigma} > 0$ and $\frac{1}{\theta} > 0$, same argument yields $\underline{k}_M > 1$ and $\underline{k}_R > 1$ if $\sigma c_0^{\sigma-1} f \geq 1$.

### A.4 Effect of $\alpha$ on the Platform-augmented Demand Component

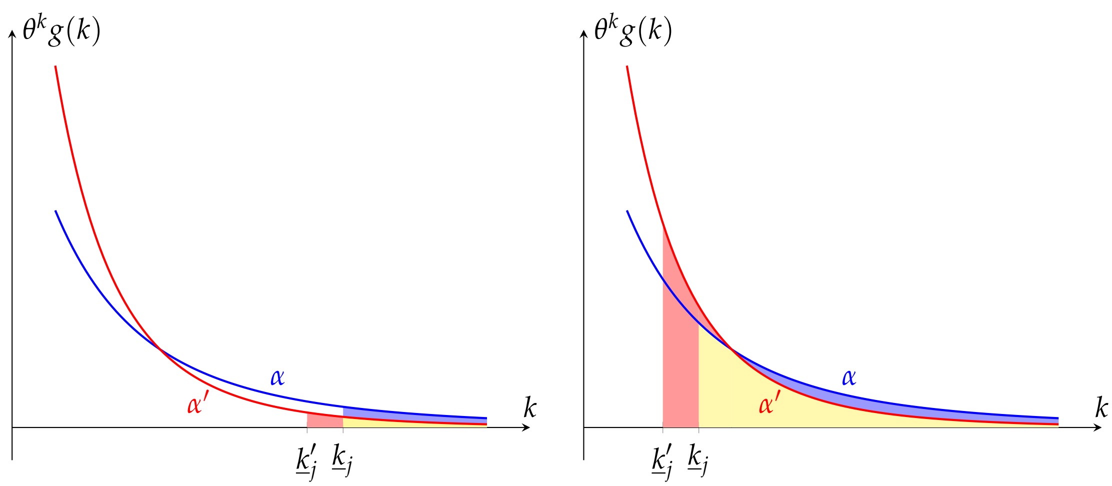


Figure 2: Effect of $\alpha$ on the Platform-augmented Demand Component

Suppose $\alpha' > \alpha$, then by Proposition 2 we know the cutoffs $\underline{k}'_j < \underline{k}_j$. In both figures, the sum of the area of the blue and yellow regions is equal to the platform-augmented demand component $\int_{\underline{k}_j}^{\infty} k^\theta \, \mathrm{d}G(k,\alpha)$, and the sum of the area of the red and yellow regions is equal to the platform-augmented demand component $\int_{\underline{k}'_j}^{\infty} k^\theta \, \mathrm{d}G(k,\alpha')$ when $\alpha$ increases to $\alpha'$, where $G(k,\alpha)$ and $G(k,\alpha')$ denote the distribution functions of the firm sizes for different values of $\alpha$.

When the cutoff value is large (the left figure) the platform-augmented demand component decreases in $\alpha$, since the area of the blue region is larger than the area of the red region. In contrast, when the cutoff value is small (the right figure), the platform-augmented demand component increases in $\alpha$ because now the area of the red region is larger than the area of the blue region.

## A.5 Proof of Proposition 4

*Proof.* For the reseller, the selling price to the final consumers is

$$p_R = \frac{(\alpha\sigma - \theta)\sigma}{\alpha(\sigma-1)^2} c, \tag{8}$$

and a firm with size $k$ has demand

$$q_R(k) = k^\theta \left[ \frac{(\alpha\sigma - \theta)\sigma}{\alpha(\sigma-1)^2} c \right]^{-\sigma}.$$

Then, the total revenue of the reseller is

$$\begin{aligned}
S_R &= N \int_{\underline{k}_R}^{\infty} p_R q_R(k) \, \mathrm{d}G(k) = N \int_{\underline{k}_R} k^\theta \left[ \frac{(\alpha\sigma - \theta)\sigma}{\alpha(\sigma-1)^2} c \right]^{1-\sigma} \frac{\alpha}{k^{\alpha+1}} \, \mathrm{d}k \\
&= N \frac{\alpha}{\alpha - \theta} \left[ \frac{(\alpha\sigma - \theta)\sigma}{\alpha(\sigma-1)^2} c \right]^{1-\sigma} (\underline{k}_R)^{\theta-\alpha} \\
&= N \frac{\alpha}{\alpha - \theta} \left[ \frac{(\alpha\sigma - \theta)\sigma}{\alpha(\sigma-1)^2} c \right]^{1-\sigma} \left\{ \left[ \frac{c^{\sigma-1}\sigma^\sigma}{(\sigma-1)^{2\sigma-1}} f \right]^{\frac{1}{\theta}} \left( \frac{\alpha\sigma - \theta}{\alpha} \right)^{\frac{\sigma}{\theta}} \left( \frac{\alpha}{\alpha - \theta} \right)^{\frac{1}{\theta}} \right\}^{\theta-\alpha}.
\end{aligned}$$

For the marketplace, the selling price is

$$p_M = \frac{\alpha\sigma^2}{(\alpha\sigma - \theta)(\sigma - 1)} c, \tag{9}$$

and the quantity of demand of a firm with size $k$ is

$$q_M(k) = k^\theta \left[ \frac{\alpha\sigma^2}{(\alpha\sigma - \theta)(\sigma - 1)} c \right]^{-\sigma}.$$

So similarly, the total revenue of the marketplace is

$$\begin{aligned}
S_M &= N \int_{\underline{k}_M}^{\infty} p_M q_M(k) \, \mathrm{d}G(k) = N \int_{\underline{k}_M} k^\theta \left[ \frac{\alpha\sigma^2}{(\alpha\sigma - \theta)(\sigma - 1)} c \right]^{1-\sigma} \frac{\alpha}{k^{\alpha+1}} \, \mathrm{d}k \\
&= N \frac{\alpha}{\alpha - \theta} \left[ \frac{\alpha\sigma^2}{(\alpha\sigma - \theta)(\sigma - 1)} c \right]^{1-\sigma} (\underline{k}_M)^{\theta-\alpha} \\
&= N \frac{\alpha}{\alpha - \theta} \left[ \frac{\alpha\sigma^2}{(\alpha\sigma - \theta)(\sigma - 1)} c \right]^{1-\sigma} \left\{ \left[ \frac{c^{\sigma-1}\sigma^\sigma}{(\sigma-1)^{\sigma-1}} f \right]^{\frac{1}{\theta}} \left( \frac{\alpha\sigma}{\alpha\sigma - \theta} \right)^{\frac{\sigma}{\theta}} \right\}^{\theta-\alpha}.
\end{aligned}$$

Therefore, the relative scale of the reseller to the marketplace measured by the GMV is

$$r = \frac{S_R}{S_M} = [\sigma(\sigma-1)]^{\frac{\alpha\sigma}{\theta}-1} \left(\frac{\alpha}{\alpha\sigma-\theta}\right)^{\frac{2\alpha\sigma}{\theta}-2} \left(\frac{\alpha-\theta}{\alpha}\right)^{\frac{\alpha}{\theta}-1}.$$

Because by (8) and (9),

$$\frac{p_M}{p_R} = \sigma(\sigma-1)\left(\frac{\alpha}{\alpha\sigma-\theta}\right)^2,$$

we have

$$r^\theta = \left(\frac{p_M}{p_R}\right)^{\alpha\sigma-\theta} \left(\frac{\alpha-\theta}{\alpha}\right)^{\alpha-\theta}$$
$$\Longrightarrow \frac{\partial r^\theta}{\partial\alpha} = \left[\ln\frac{\alpha-\theta}{\alpha} + \sigma\ln\frac{p_M}{p_R} + (\alpha\sigma-\theta)\frac{p_R}{p_M}\cdot\frac{\partial(p_M/p_R)}{\partial\alpha} + \frac{\theta}{\alpha}\right] r^\theta.$$

Since

$$\frac{\partial(p_M/p_R)}{\partial\alpha} = -\frac{2\alpha\theta\sigma(\sigma-1)}{(\alpha\sigma-\theta)^3} \Longrightarrow \frac{p_R}{p_M}\cdot\frac{\partial(p_M/p_R)}{\partial\alpha} = -\frac{2\theta}{\alpha(\alpha\sigma-\theta)},$$

we further have

$$\frac{\partial r^\theta}{\partial\alpha} = \left(\ln\frac{\alpha-\theta}{\alpha} + \sigma\ln\frac{p_M}{p_R} - \frac{\theta}{\alpha}\right) r^\theta.$$

Given $\ln\frac{\alpha-\theta}{\alpha} < 0$, condition (5) implies $p_M/p_R < 1$ by Lemma 1, then $r$ is decreasing in $\alpha$ since

$$\frac{\partial r^\theta}{\partial\alpha} < 0 \Longrightarrow \frac{\partial r}{\partial\alpha} < 0. \qquad \square$$

## A.6 Example of Products

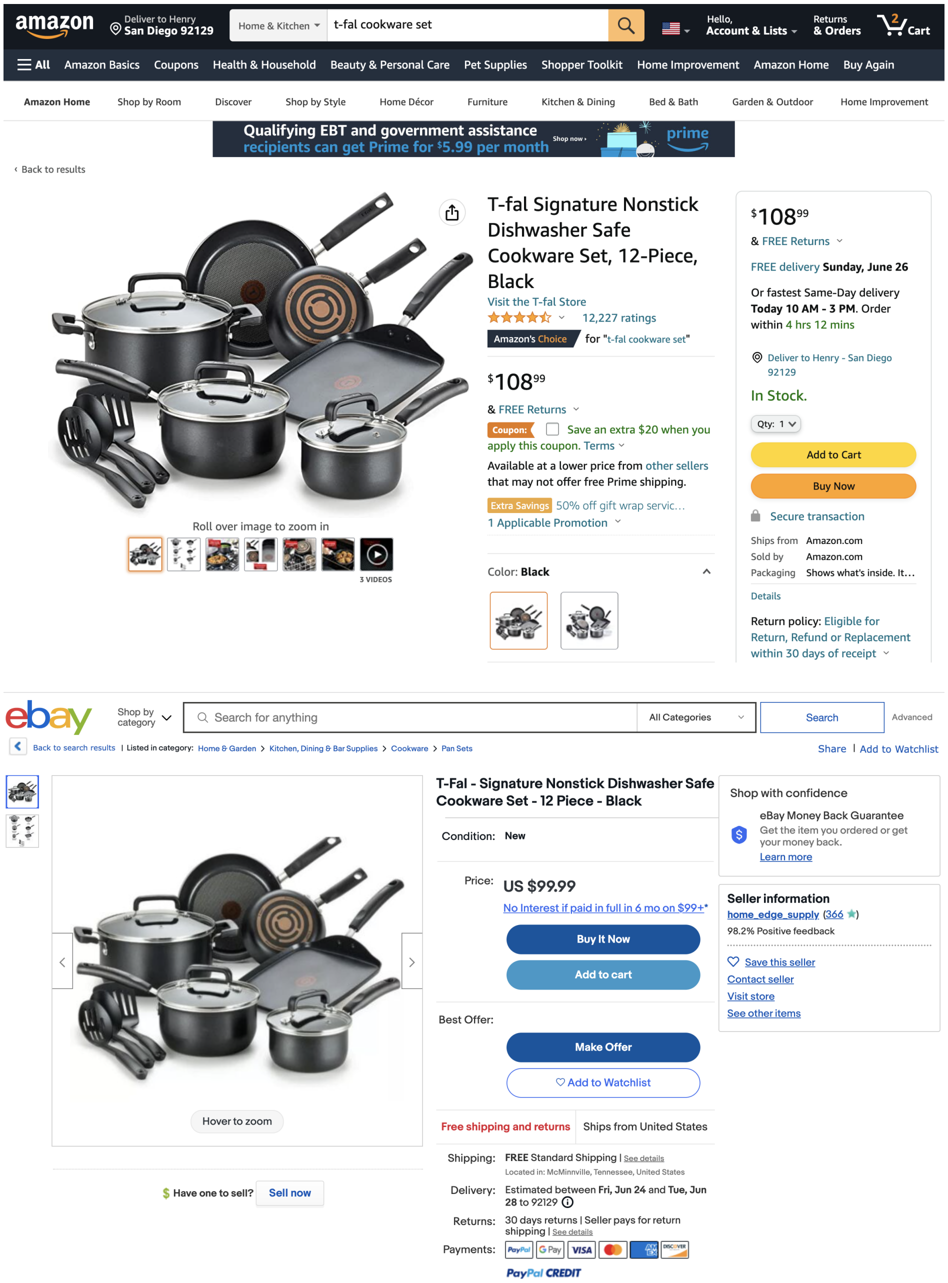


Figure 3: An Example of Identical Products Sold on Amazon and eBay

## A.7 Extended Model: Size-Dependent Marginal Cost

### A.7.1 Marketplace

**Firm:** Given $\lambda \in (0,1)$, a firm maximizes profit

$$\pi_M(k) = (1-\lambda)pq - cq - f = (1-\lambda)k^\theta p^{1-\sigma} - c_0 p^{-\sigma} - f.$$

The first order condition yields:

$$p = \frac{c\sigma}{(1-\lambda)(\sigma-1)} = \frac{c_0 k^{-\theta}\sigma}{(1-\lambda)(\sigma-1)}.$$

Note that the price has the same expression as the previous case, but now it is indeed size-dependent because the marginal cost $c = c_0 k^{-\theta}$ varies with firm sizes.

Thus, the firm's demand is

$$q = k^\theta p^{-\sigma} = \left[\frac{c_0\sigma}{(1-\lambda)(\sigma-1)}\right]^{-\sigma} k^{\theta(\sigma+1)},$$

and the profit is

$$\pi_M(k) = k^{\theta\sigma}\frac{c_0^{1-\sigma}\sigma^{-\sigma}}{(1-\lambda)^{-\sigma}(\sigma-1)^{1-\sigma}} - f.$$

Therefore, a firm chooses to sell via marketplace if $\pi_M(k) \geq 0$. It gives the firm size cutoff

$$\underline{k}_M = \left(\frac{1}{1-\lambda}\right)^{\frac{1}{\theta}} \left[\frac{\sigma^\sigma c_0^{\sigma-1} f}{(\sigma-1)^{\sigma-1}}\right]^{\frac{1}{\theta\sigma}}$$

**Marketplace:** Given only firms with size at least $\underline{k}_M$ sell via the marketplace, we have the profit of marketplace

$$\begin{aligned}\Pi_M = N\int_{\underline{k}_M}^{\infty} \lambda p(k) q(p(k),k)\,\mathrm{d}G(k) &= N\int_{\underline{k}_M}^{\infty} \lambda k^{\theta\sigma}\left[\frac{c_0\sigma}{(1-\lambda)(\sigma-1)}\right]^{1-\sigma}\frac{\alpha}{k^{\alpha+1}}\,\mathrm{d}k \\ &= N\frac{\alpha\lambda}{\alpha-\theta\sigma}\left[\frac{c_0\sigma}{(1-\lambda)(\sigma-1)}\right]^{1-\sigma}(\underline{k}_M)^{\theta\sigma-\alpha},\end{aligned}$$

provided that $\theta\sigma < \alpha$. The marketplace maximizes its profit $\Pi_M$ by choosing the proportion $\lambda$. The first order condition gives

$$\lambda = \frac{\theta}{\alpha}.$$

Then the selling price of firm with size $k$ is

$$p_M(k) = \frac{c_0 k^{-\theta}\sigma}{(1-\lambda)(\sigma-1)} = \frac{\alpha\sigma}{(\alpha-\theta)(\sigma-1)} c_0 k^{-\theta},$$

and the cutoff is

$$\underline{k}_M = \left(\frac{\alpha}{\alpha-\theta}\right)^{\frac{1}{\theta}} \left[\frac{\sigma^\sigma c_0^{\sigma-1} f}{(\sigma-1)^{\sigma-1}}\right]^{\frac{1}{\theta\sigma}}.$$

Finally, given $\alpha > \theta\sigma$, the total revenue or the GMV of the marketplace is

$$\begin{aligned} S_M &= N\int_{\underline{k}_M}^{\infty} p_M(k) q(p_M(k), k)\, \mathrm{d}G(k) = N\int_{\underline{k}_M}^{\infty} k^\theta \left[c_0 k^{-\theta} \frac{\alpha\sigma}{(\alpha-\theta)(\sigma-1)}\right]^{1-\sigma} \cdot \frac{\alpha}{k^{\alpha+1}}\, \mathrm{d}k \\ &= N\left(\frac{c_0\sigma}{\sigma-1}\right)^{1-\sigma} \frac{\alpha^{2-\sigma}}{(\alpha-\theta)^{1-\sigma}} \cdot \frac{(\underline{k}_M)^{\theta\sigma-\alpha}}{\alpha-\theta\sigma} = N\frac{\alpha^{2-\frac{\alpha}{\theta}} \sigma^{1-\frac{\alpha}{\theta}} c_0^{\frac{\alpha}{\theta\sigma}-\frac{\alpha}{\theta}} f^{1-\frac{\alpha}{\theta\sigma}}}{(\alpha-\theta)^{1-\frac{\alpha}{\theta}}(\alpha-\theta\sigma)(\sigma-1)^{\frac{\alpha}{\theta\sigma}-\frac{\alpha}{\theta}}}. \end{aligned}$$

### A.7.2 Reseller

Let $\mathcal{K} \subset (1, +\infty)$ be the collection of firms selling via reseller. The profit of the reseller is

$$\Pi_R = N\int_{\mathcal{K}} (p-s) q(p,k)\, \mathrm{d}G(k) = N\int_{\mathcal{K}} (p-s) k^\theta p^{-\sigma}\, \mathrm{d}G(k).$$

Given the purchase price $s$, $\mathcal{K}$ is fixed, so the first order condition of setting selling price gives

$$p = \frac{\sigma}{\sigma-1} s.$$

**Firm:** Given the purchase price $s$, a firm with size $k$ has demand

$$q = k^\theta p^{-\sigma} = k^\theta \left(\frac{\sigma s}{\sigma-1}\right)^{-\sigma},$$

and the profit of the firm is

$$\pi_R(k) = (s-c)q - f = (s - c_0 k^{-\theta}) k^\theta \left(\frac{\sigma s}{\sigma-1}\right)^{-\sigma} - f.$$

The firm chooses to sell via reseller if $\pi_R(k) \geq 0$, i.e.

$$s k^\theta - c_0 \geq \left(\frac{\sigma s}{\sigma-1}\right)^\sigma f \implies k \geq \left[\left(\frac{\sigma s}{\sigma-1}\right)^\sigma \frac{f}{s} + \frac{c_0}{s}\right]^{\frac{1}{\theta}} \equiv \underline{k}_R.$$

**Reseller:** Given only firms with size at least $\underline{k}_R$ sell via the reseller, the profit of reseller is

$$\Pi_R = N \int_{\underline{k}_R}^{\infty} (p-s) q(p,k) \,\mathrm{d}G(k) = N \int_{\underline{k}_R}^{\infty} \left( \frac{\sigma}{\sigma-1} - 1 \right) s k^{\theta} \left( \frac{\sigma s}{\sigma - 1} \right)^{-\sigma} \frac{\alpha}{k^{\alpha+1}} \,\mathrm{d}k$$
$$= N \frac{\alpha \sigma^{-\sigma} s^{1-\sigma}}{(\alpha-\theta)(\sigma-1)^{1-\sigma}} (\underline{k}_R)^{\theta-\alpha}.$$

First order condition of reseller's profit maximization yields

$$s = \frac{\sigma-1}{\sigma} \left( \frac{\alpha - \theta\sigma}{\alpha\sigma - \alpha} \cdot \frac{c_0}{f} \right)^{\frac{1}{\sigma}}$$

and then the selling price of the reseller

$$p_R = \frac{\sigma}{\sigma-1} s = \left( \frac{\alpha - \theta\sigma}{\alpha\sigma - \alpha} \cdot \frac{c_0}{f} \right)^{\frac{1}{\sigma}}.$$

So the firm size cutoff is

$$\underline{k}_R = \left[ \left( \frac{\alpha - \theta\sigma}{\alpha\sigma - \alpha} \cdot \frac{c_0}{f} \cdot f + c_0 \right) \frac{\sigma}{\sigma-1} \left( \frac{\alpha - \theta\sigma}{\alpha\sigma - \alpha} \cdot \frac{c_0}{f} \right)^{-\frac{1}{\sigma}} \right]^{\frac{1}{\theta}}$$
$$= \left[ \frac{(\alpha-\theta)(\alpha-\theta\sigma)^{-\frac{1}{\sigma}}}{\alpha^{1-\frac{1}{\sigma}}} \right]^{\frac{1}{\theta}} \left[ \frac{\sigma^{2\sigma} c_0^{\sigma-1} f}{(\sigma-1)^{2\sigma-1}} \right]^{\frac{1}{\theta\sigma}},$$

and the total revenue or the GMV for the reseller is

$$S_R = N \int_{\underline{k}_R}^{\infty} p_R q(p_R, k) \,\mathrm{d}G(k) = N \int_{\underline{k}_R}^{\infty} k^{\theta} \left( \frac{\alpha - \theta\sigma}{\alpha\sigma - \alpha} \cdot \frac{c_0}{f} \right)^{\frac{1-\sigma}{\sigma}} \frac{\alpha}{k^{\alpha+1}} \,\mathrm{d}k$$
$$= N\alpha \left( \frac{\alpha - \theta\sigma}{\alpha\sigma - \alpha} \cdot \frac{c_0}{f} \right)^{\frac{1-\sigma}{\sigma}} \frac{(\underline{k}_R)^{\theta-\alpha}}{\alpha-\theta} = N \frac{\alpha^{1+\frac{\alpha}{\theta}-\frac{\alpha}{\theta\sigma}} \sigma^{2-2\frac{\alpha}{\theta}} c_0^{\frac{\alpha}{\theta\sigma}-\frac{\alpha}{\theta}} f^{1-\frac{\alpha}{\theta\sigma}}}{(\alpha-\theta)^{\frac{\alpha}{\theta}} (\alpha-\theta\sigma)^{1-\frac{\alpha}{\theta\sigma}} (\sigma-1)^{\frac{\alpha}{\theta\sigma}-\frac{2\alpha}{\theta}+1}}.$$

## A.8 Proof of Proposition 6

*Proof.* Given the scales $S_R$ and $S_M$ of the two platforms, the relative scale

$$r = \frac{S_R}{S_M} = \left( \frac{\sigma-1}{\sigma} \right)^{\frac{\alpha}{\theta}-1} \left( \frac{\alpha}{\alpha-\theta} \right)^{2\frac{\alpha}{\theta}-1} \left( \frac{\alpha-\theta\sigma}{\alpha} \right)^{\frac{\alpha}{\theta\sigma}}.$$

So

$$\frac{\partial \ln r}{\partial \alpha} = \frac{\partial}{\partial \alpha} \left[ \left( \frac{\alpha}{\theta} - 1 \right) \ln \frac{\sigma-1}{\sigma} + \left( \frac{2\alpha}{\theta} - 1 \right) \ln \frac{\alpha}{\alpha-\theta} + \frac{\alpha}{\theta\sigma} \ln \frac{\alpha-\theta\sigma}{\alpha} \right]$$

$$= \frac{1}{\theta}\left(\ln\frac{\sigma-1}{\sigma} - 2\ln\frac{\alpha-\theta}{\alpha} + \frac{1}{\sigma}\ln\frac{\alpha-\theta\sigma}{\alpha}\right) - \frac{2\alpha-\theta}{\alpha(\alpha-\theta)} + \frac{1}{\alpha-\theta\sigma}$$
$$= \frac{1}{\theta}\ln\left[\left(1-\frac{1}{\sigma}\right)\left(1-\frac{\theta}{\alpha}\right)^{-2}\left(1-\frac{\theta/\alpha}{1/\sigma}\right)^{\frac{1}{\sigma}}\right] - \frac{\alpha\sigma}{(\alpha-\theta)(\alpha-\theta\sigma)}\left[\frac{1}{\sigma} - 2\frac{\theta}{\alpha} + \left(\frac{\theta}{\alpha}\right)^2\right].$$

Consider reparametrization $u = 1/\sigma$ and $v = \theta/\alpha$, and define functions

$$\varphi_1(u,v) = \frac{(1-u)(1-v/u)^u}{(1-v)^2}, \quad \varphi_2(u,v) = u - 2v + v^2.$$

Then we can rewrite $\partial \ln r/\partial\alpha$ as

$$\frac{\partial \ln r}{\partial \alpha} = \frac{\ln \varphi_1(u,v)}{\theta} - \frac{\alpha\sigma}{(\alpha-\theta)(\alpha-\theta\sigma)}\varphi_2(u,v).$$

Therefore, $\partial \ln r/\partial\alpha < 0$ if $\varphi_1(u,v) < 1$ and $\varphi_2(u,v) > 0$.

Note that because $\sigma > 1$ and $\alpha > \theta\sigma > \theta$, the new parameters $0 < v < u < 1$. For every fixed $u \in (0,1)$,

$$\frac{\partial \varphi_1(u,v)}{\partial v} = \frac{(1-u)(1-v/u)^u}{(u-v)(1-v)^3}[u + (u-2)v].$$

Then

$$\frac{\partial \varphi_1(u,v)}{\partial v} = 0 \implies u + (u-2)v = 0 \implies v = \frac{u}{2-u} < u$$

because $2-u > 1$. We further have $\partial\varphi_1/\partial v > 0$ for $0 < v < \frac{u}{2-u}$ and $\partial\varphi_1/\partial v < 0$ for $\frac{u}{2-u} < v < u$, which implies that $v = \frac{u}{2-u}$ maximizes $\varphi_1(u,v)$ for any given $0 < u < 1$. Next,

$$\varphi_1\left(u, \frac{u}{2-u}\right) = \frac{(1-u)\left(1-\frac{1}{2-u}\right)^u}{\left(1-\frac{u}{2-u}\right)^2} = \frac{2-u}{4}\left(1-\frac{1}{2-u}\right)^{u-1}$$
$$\implies \frac{\partial \varphi_1\left(u, \frac{u}{2-u}\right)}{\partial u} = \frac{(2-u)^2}{4(1-u)}\left(1-\frac{1}{2-u}\right)^u \ln\frac{1-u}{2-u} < 0 \quad \text{for all } 0 < u < 1$$

because $\frac{1-u}{2-u} < 1$. This means $\varphi_1\left(u, \frac{u}{2-u}\right)$ is strictly decreasing in $u$, and therefore for any $0 < v < u < 1$,

$$\varphi_1(u,v) \le \varphi_1\left(u, \frac{u}{2-u}\right) < \lim_{u\to 0}\varphi_1\left(u, \frac{u}{2-u}\right) = 1.$$

Meanwhile, it can be verified that

$$\varphi_2(u,v) = (1-v)^2 - (1-u) > 0 \quad \text{if } v < 1 - \sqrt{1-u}.$$

Because $1 - \sqrt{1-u} < 1 - (1-u) = u$, combining the above results, we have $\varphi_1(u,v) < 1$ and

$\varphi_2(u,v) > 0$ if $0 < v < 1 - \sqrt{1-u}$, i.e.

$$\frac{\theta}{\alpha} < 1 - \sqrt{1 - \frac{1}{\sigma}} = 1 - \frac{\sqrt{\sigma(\sigma-1)}}{\sigma} \quad \text{or equivalently} \quad \alpha > \frac{\theta\sigma}{\sigma - \sqrt{\sigma(\sigma-1)}}.$$

Hence, when $\alpha > \frac{\theta\sigma}{\sigma-\sqrt{\sigma(\sigma-1)}}$ and $\sigma > 1$, $\partial \ln r/\partial\alpha < 0 \implies \partial r/\partial\alpha < 0$. $\square$

## References


ABHISHEK, V., K. JERATH, AND Z. J. ZHANG (2016): "Agency Selling or Reselling? Channel Structures in Electronic Retailing," *Management Science*, 62, 2259–2280.

ALFARO, L., A. CHARLTON, AND F. KANCZUK (2009): "Plant Size Distribution and Cross-Country Income Differences," *NBER International Seminar on Macroeconomics*, 5, 243–272.

ARMSTRONG, M. (2006): "Competition in Two-Sided Markets," *RAND Journal of Economics*, 37, 669–691.

AXTELL, R. L. (2001): "Zipf Distribution of U.S. Firm Sizes," *Science*, 293, 1818–1820.

BAKOS, J. Y. (1997): "Reducing Buyer Search Costs: Implications for Electronic Marketplaces," *Management Science*, 43, 1676–1692.

BARTELSMAN, E., J. HALTIWANGER, AND S. SCARPETTA (2013): "Cross-country Differences in Productivity: The Role of Allocation and Selection," *American Economic Review*, 103, 305–334.

BENTO, P. AND D. RESTUCCIA (2017): "Misallocation, Establishment Size, and Productivity," *American Economic Journal: Macroeconomics*, 9, 267–303.

——— (2021): "On Average Establishment Size across Sectors and Countries," *Journal of Monetary Economics*, 117, 220–242.

BOUDREAU, K. J. (2017): "Platform Boundary Choices & Governance: Opening-up While Still Coordinating and Orchestrating," in *Entrepreneurship, Innovation, and Platforms*, Emerald Publishing Limited, vol. 37 of *Advances in Strategic Management*, 227–297.

BROWN, J. R. AND A. GOOLSBEE (2002): "Does the Internet Make Markets More Competitive? Evidence from the Life Insurance Industry," *Journal of Political Economy*, 110, 481–507.

BRYNJOLFSSON, E. AND M. D. SMITH (2000): "Frictionless Commerce? A Comparison of Internet and Conventional Retailers," *Management Science*, 46, 563–585.

CAILLAUD, B. AND B. JULLIEN (2003): "Chicken & Egg: Competition among Intermediation Service Providers," *RAND Journal of Economics*, 34, 309–328.

EINAV, L., C. FARRONATO, AND J. LEVIN (2016): "Peer-to-Peer Markets," *Annual Review of Economics*, 8, 615–635.

FARRONATO, C. (2018): "Pricing Mechanisms in Online Markets," in *The New Palgrave Dictionary of Economics*, London: Palgrave Macmillan UK.

GABAIX, X. (2009): "Power Laws in Economics and Finance," *Annual Review of Economics*, 1, 255–259.

GOLDMANIS, M., A. HORTAÇSU, C. SYVERSON, AND O. EMRE (2010): "E-commerce and the Market Structure of Retail Industries," *Economic Journal*, 120, 651–682.

GUNER, N., G. VENTURA, AND Y. XU (2008): "Macroeconomic Implications of Size-dependent Policies," *Review of Economic Dynamics*, 11, 721–744.

HAGIU, A. (2007): "Merchant or Two-Sided Platform," *Review of Network Economics*, 6, 115–133.

HAGIU, A. AND R. S. LEE (2011): "Exclusivity and Control," *Journal of Economics Management and Strategy*, 20, 679–708.

HAGIU, A. AND J. WRIGHT (2015): "Marketplace or Reseller?" *Management Science*, 61, 184–205.

HSIEH, C.-T. AND P. J. KLENOW (2009): "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, 124, 1403–1448.

HSIEH, C.-T. AND B. A. OLKEN (2014): "The Missing "Missing Middle"," *Journal of Economic Perspectives*, 28, 89–108.

JIN, G. Z. AND A. KATO (2007): "Dividing Online and Offline: A Case Study," *Review of Economic Studies*, 74, 9819–1004.

KALEMLI-OZCAN, S., B. SORENSEN, C. VILLEGAS-SANCHEZ, V. VOLOSOVYCH, AND S. YESILTAS (2015): "How to Construct Nationally Representative Firm Level Data from the Orbis Global Database: New Facts and Aggregate Implications," Working Paper 21558, National Bureau of Economic Research.

LIEBER, E. AND C. SYVERSON (2012): "Online versus Offline Competition," in *The Oxford Handbook of the Digital Economy*, ed. by M. Peitz and J. Waldfogel, Oxford University Press.

POSCHKE, M. (2018): "The Firm Size Distribution across Countries and Skill-Biased Change in Entrepreneurial Technology," *American Economic Journal: Macroeconomics*, 10, 1–41.

ROCHET, J.-C. AND J. TIROLE (2006): "Two-Sided Markets: A Progress Report," *RAND Journal of Economics*, 35, 645–667.

RYSMAN, M. (2009): "The Economics of Two-Sided Markets," *Journal of Economic Perspectives*, 23, 125–143.

SUTTON, J. (1997): "Gibrat's Legacy," *Journal of Economy Literature*, 35, 40–59.

TYBOUT, J. R. (2000): "Manufacturing Firms in Developing Countries: How Well Do They Do, and Why?" *Journal of Economic Literature*, 38, 11–44.

WEYL, E. G. (2010): "A Price Theory of Multi-Sided Platforms," *American Economic Review*, 100, 1642–1672.