Supplement to "Belief Error and Non-Bayesian Social Learning: An Experimental Evidence"

Boğaçhan Çelen[1], Sen Geng[2], and Huihui Li[2]

[1]University of Melbourne
[2]Xiamen University

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This supplement contains two parts. Appendix B contains proofs and lengthy derivations omitted from the main body of the paper. Appendix C provides the translation of the Experimental Instructions presented to the subjects.

B Supplementary Results

B.1 Proof of Proposition 2

Proof. Without loss of generality, we assume that an agent is endowed with a signal $s_0 = B$. It is clear that this posterior about state 1 becomes p, which implies that her optimal choice is action 1 and the expected payoff is p.

When observing an additional signal, her posterior about state 1 may be $\frac{p^2}{p^2+q^2}$ or $\frac{1}{2}$ depending on the realization of $s_1 = B$ or $s_1 = W$. In the former case, her optimal choice is action 1 and the expected payoff is $\frac{p^2}{p^2+q^2}$; in the latter case, she is indifferent from choosing either action and the expected payoff is $\frac{1}{2}$. Then her ex-ante expected payoff from observing an additional signal will be

$$p \cdot \left(p \cdot \frac{p^2}{p^2+q^2} + q \cdot \frac{1}{2} \right) + q \cdot \left(q \cdot \frac{p^2}{p^2+q^2} + p \cdot \frac{1}{2} \right) = p,$$

which is the same as her expected payoff based on a signal s_0. So there is no informational value of observing an additional signal, i.e., $W(p, 1) = 0$.

Suppose now the agent decides to observe $2n$ additional signals ($n \geqslant 1$). We first notice that if the agent suboptimally choose action 1 regardless of the realizations of the $2n$ additional signals, her expected payoff from observing these signals, denoted as $\mathbb{E}(U_{\text{act1}} \mid s_0)$ will still be p. Specifically,

$$\begin{aligned}
\mathbb{E}(U_{\text{act1}} \mid s_0) &= P(\text{state } 1 \mid s_0) \cdot \mathbb{E}(U_{\text{act1}} \mid \text{state } 1, s_0) + P(\text{state } 2 \mid s_0) \cdot \mathbb{E}(U_{\text{act1}} \mid \text{state } 2, s_0) \\
&= p \left(C_{2n}^{0} p^{2n} \cdot \frac{p^{2n+1}}{p^{2n+1} + q^{2n+1}} + \cdots + C_{2n}^{2n} q^{2n} \cdot \frac{pq^{2n}}{pq^{2n} + p^{2n}q} \right) \\
&\qquad + q \left(C_{2n}^{0} q^{2n} \cdot \frac{p^{2n+1}}{p^{2n+1} + q^{2n+1}} + \cdots + C_{2n}^{2n} p^{2n} \cdot \frac{pq^{2n}}{pq^{2n} + p^{2n}q} \right) \\
&= p \left(C_{2n}^{0} p^{2n} + C_{2n}^{1} p^{2n-1} q + \cdots + C_{2n}^{2n} q^{2n} \right) \\
&= p \cdot (p+q)^{2n} \\
&= p.
\end{aligned}$$

So the increment in her expected payoff from the additional observations, if any, must come from action switch, i.e., from those observations based on which action 2 delivers a higher expected payoff than action 1. Due to the symmetry of the signal structure, we know that the agent should optimally choose action 2 if and only if the $2n$ additional signals include at least $n+1$ signals with value W. Consider a case in which there are exactly k signals with value W, where $n+1 \leqslant k \leqslant 2n$. Compared to $\mathbb{E}(U_{\text{act1}} \mid s_0)$, the increase in the expected payoff from this case should be

$$\begin{aligned}
&(p \cdot p^{2n-k} q^k + q \cdot q^{2n-k} p^k) \cdot \frac{p^k q^{2n+1-k}}{p^{2n+1-k} q^k + p^k q^{2n+1-k}} \\
&\qquad - (p \cdot p^{2n-k} q^k + q \cdot q^{2n-k} p^k) \cdot \frac{p^{2n+1-k} q^k}{p^{2n+1-k} q^k + p^k q^{2n+1-k}} = p^k q^{2n+1-k} - p^{2n+1-k} q^k.
\end{aligned}$$

Since there are C_{2n}^{k} scenarios in which exactly k signals have a realization of W, $W(p, 2n) = \sum_{k=n+1}^{2n} C_{2n}^{k} \left(p^k q^{2n+1-k} - p^{2n+1-k} q^k \right)$.

We now establish that the expected payoff from observing $2n+1$ additional signals ($n \geqslant 1$) is identical to that from observing $2n$ additional signals. When there are $2n+1$ additional signals, the agent should optimally choose action 2 if and only if the $2n+1$ additional signals include at least $n+2$ signals with value W.[1] So the same derivation gives $W(p, 2n+1) = \sum_{k=n+2}^{2n+1} C_{2n+1}^{k} \left(p^k q^{2n+2-k} - p^{2n+2-k} q^k \right)$. Therefore, using

[1] When there are $n+1$ signals of value W, action 1 and action 2 yield the same expected payoff.

$C^k_{2n+1} = C^k_{2n} + C^{k-1}_{2n}$ for $n+2 \leqslant k \leqslant 2n$, we have

$$
\begin{aligned}
&W(p,2n+1) - W(p,2n) \\
&\quad = \left[\left(p^{2n+1}q - pq^{2n+1}\right) - \left(p^{2n}q - pq^{2n}\right)\right] \\
&\qquad + \sum_{k=n+2}^{2n} \left[\left(C^k_{2n} + C^{k-1}_{2n}\right)\left(p^k q^{2n+2-k} - p^{2n+2-k}q^k\right) - C^{k-1}_{2n}\left(p^{k-1}q^{2n+2-k} - p^{2n+2-k}q^{k-1}\right)\right] \\
&\quad = \sum_{k=n+2}^{2n} C^k_{2n}\left(p^k q^{2n+2-k} - p^{2n+2-k}q^k\right) + C^{2n}_{2n}\left[\left(p^{2n+1}q - pq^{2n+1}\right) - \left(p^{2n}q - pq^{2n}\right)\right] \\
&\qquad\quad + \sum_{k=n+2}^{2n-1} C^k_{2n}\left[\left(p^{k+1}q^{2n+1-k} - p^{2n+1-k}q^{k+1}\right) - \left(p^k q^{2n+1-k} - p^{2n+1-k}q^k\right)\right] \\
&\qquad\qquad + C^{n+1}_{2n}\left[(p^{n+2}q^n - p^n q^{n+2}) - (p^{n+1}q^n - p^n q^{n+1})\right] \\
&\quad = \sum_{k=n+2}^{2n} C^k_{2n}(p+q-1)\left(p^k q^{2n+1-k} - p^{2n+1-k}q^k\right) + C^{n+1}_{2n}\left[(p-1)p^{n+1}q^n - (q-1)p^n q^{n+1}\right].
\end{aligned}
$$

Since $p+q=1$, each term in the above summation is zero and then

$$
W(p,2n+1) - W(p,2n) = C^{n+1}_{2n}\left(p^{n+1}q^{n+1} - p^{n+1}q^{n+1}\right) = 0.
$$

We know that $W(p,2n+1) = \sum_{k=n+2}^{2n+1} C^k_{2n+1}\left(p^k q^{2n+2-k} - p^{2n+2-k}q^k\right)$ and, by similar derivation, $W(p,2n+2) = \sum_{k=n+2}^{2n+2} C^k_{2n+2}\left(p^k q^{2n+3-k} - p^{2n+3-k}q^k\right)$. Applying the algebra operation used to show property (i), we have

$$
\begin{aligned}
&W(p,2n+2) - W(p,2n+1) \\
&\quad = C^{n+2}_{2n+2}\left(p^{n+2}q^{n+1} - p^{n+1}q^{n+2}\right) \\
&\qquad + \left[\sum_{k=n+3}^{2n+2} C^k_{2n+2}\left(p^k q^{2n+3-k} - p^{2n+3-k}q^k\right) - \sum_{k=n+2}^{2n+1} C^k_{2n+1}\left(p^k q^{2n+2-k} - p^{2n+2-k}q^k\right)\right] \\
&\quad = \left(C^{n+2}_{2n+1} + C^{n+1}_{2n+1}\right)\left(p^{n+2}q^{n+1} - p^{n+1}q^{n+2}\right) + C^{n+2}_{2n+1}\left[(p-1)p^{n+2}q^n - (q-1)p^n q^{n+2}\right] \\
&\qquad\qquad + \sum_{k=n+3}^{2n+1} C^k_{2n+1}(p+q-1)\left(p^k q^{2n+2-k} - p^{2n+2-k}q^k\right) \\
&\quad = C^{n+1}_{2n+1}(p-q)p^{n+1}q^{n+1} > 0
\end{aligned}
$$

as $p > q$. This establishes that $W(p,2(n+1)) > W(p,2n+1) = W(p,2n)$.

To establish the limit property, note that $p = p(p+q)^{2n} = \sum_{k=0}^{2n} C^k_{2n} p^{2n+1-k} q^k$. This implies that

$$\begin{aligned}
W(p,2n)+p &= \sum_{k=n+1}^{2n} C_{2n}^{k} p^{k} q^{2n+1-k} + \sum_{k=0}^{n} C_{2n}^{k} p^{2n+1-k} q^{k} \\
&= \left(\sum_{k=n+1}^{2n} C_{2n}^{k} p^{k} q^{2n+1-k} + \sum_{k=n+1}^{2n} C_{2n}^{2n-k} p^{k+1} q^{2n-k} \right) + C_{2n}^{n} p^{n+1} q^{n} \\
&= \sum_{k=n+1}^{2n} C_{2n}^{k} p^{k} q^{2n-k} (p+q) + C_{2n}^{n} p^{n+1} q^{n} \\
&= 1 - \sum_{k=0}^{n} C_{2n}^{k} p^{k} q^{2n-k} + C_{2n}^{n} p^{n+1} q^{n}.
\end{aligned}$$

Since $n < 2n \cdot p$, Hoeffding's inequality implies that $\sum_{k=0}^{n} C_{2n}^{k} p^{k} q^{2n-k} \leqslant \exp\left[-4n\left(p-\frac{1}{2}\right)^2\right]$. So as $n \to \infty$, $\sum_{k=0}^{n} C_{2n}^{k} p^{k} q^{2n-k} \to 0$ and also $C_{2n}^{n} p^{n+1} q^{n} \to 0$. As a result, $W(p,2n)+1 \to p$, i.e., $\lim_{n\to\infty} W(p,2n) = 1-p$.

We now prove properties (iii) and (v). Note that $kC_{2n}^{k} = 2nC_{2n-1}^{k-1}$ for $1 \leqslant k \leqslant 2n$ and $(2n-k)C_{2n}^{k} = 2nC_{2n-1}^{k}$ for $0 \leqslant k \leqslant 2n-1$, then

$$\begin{aligned}
\frac{\partial W(p,2n)}{\partial p} &= \sum_{k=n+1}^{2n} C_{2n}^{k} \left[k\left(p^{k-1}q^{2n+1-k} + p^{2n+1-k}q^{k-1}\right) - (2n+1-k)\left(p^{k}q^{2n-k} + p^{2n-k}q^{k}\right) \right] \\
&= \sum_{k=n+1}^{2n} 2nC_{2n-1}^{k-1}\left(p^{k-1}q^{2n+1-k} + p^{2n+1-k}q^{k-1}\right) - \sum_{k=n+1}^{2n-1} 2nC_{2n-1}^{k}\left(p^{k}q^{2n-k} + p^{2n-k}q^{k}\right) \\
&\qquad - \sum_{k=n+1}^{2n} C_{2n}^{k}\left(p^{k}q^{2n-k} + p^{2n-k}q^{k}\right) \\
&= 2n\left[\sum_{k=n}^{2n-1} C_{2n-1}^{k}\left(p^{k}q^{2n-k} + p^{2n-k}q^{k}\right) - \sum_{k=n+1}^{2n-1} C_{2n-1}^{k}\left(p^{k}q^{2n-k} + p^{2n-k}q^{k}\right) \right] \\
&\qquad - \sum_{k=0}^{n-1} C_{2n}^{k} p^{k} q^{2n-k} - \sum_{k=n+1}^{2n} C_{2n}^{k} p^{k} q^{2n-k} \\
&= 2nC_{2n-1}^{n} \cdot 2p^{n}q^{n} - \left(1 - C_{2n}^{n} p^{n} q^{n}\right) \\
&= (2n+1)C_{2n}^{n} p^{n} q^{n} - 1.
\end{aligned}$$

It immediately follows that $\partial^2 W(p,2n)/\partial p^2 = (2n+1)np^{n-1}q^{n-1}(q-p) < 0$ for $p > \frac{1}{2}$. When $p=1$, $\partial W(p,2n)/\partial p = -1 < 0$, and when $p = \frac{1}{2}$,

$$\frac{\partial W(p,2n)}{\partial p} = \frac{(2n+1)C_{2n}^{n}}{4^n} - 1 = \frac{3 \cdot 5 \cdot \cdots \cdot (2n+1)}{2 \cdot 4 \cdot \cdots \cdot 2n} - 1 > 0.$$

So there exists a unique $p^*_{2n} \in \left(\frac{1}{2}, 1\right)$ such that $\partial W(p,2n)/\partial p = 0$ when $p = p^*_{2n}$. When $\frac{1}{2} \leqslant p < p^*_{2n}$, $\partial W(p,2n)/\partial p > 0$ so $W(p,2n)$ is increasing in p; when $p^*_{2n} < p \leqslant 1$, $\partial W(p,2n)/\partial p < 0$ so $W(p,2n)$ is decreasing in p. We can solve the threshold p^*_{2n} as

$$p^*_{2n} = \frac{1}{2} + \frac{1}{2}\left[1 - \frac{4}{\sqrt[n]{(2n+1)C^n_{2n}}}\right]^{\frac{1}{2}} = \frac{1}{2} + \frac{1}{2}\left[1 - \left(\frac{2 \cdot 4 \cdot \cdots \cdot 2n}{3 \cdot 5 \cdot \cdots \cdot (2n+1)}\right)^{\frac{1}{n}}\right]^{\frac{1}{2}}.$$

We now prove property (iv). Let $k_n = \sqrt[n]{(2 \cdot 4 \cdot \cdots \cdot 2n)/(3 \cdot 5 \cdot \cdots \cdot (2n+1))}$. Note that

$$\begin{aligned}(k_{n+1})^{n(n+1)} &= \left[\frac{2 \cdot 4 \cdot \cdots \cdot (2n) \cdot (2n+2)}{3 \cdot 5 \cdot \cdots \cdot (2n+1) \cdot (2n+3)}\right]^n = \left(\frac{2n+2}{2n+3}\right)^n \left[\frac{2 \cdot 4 \cdot \cdots \cdot (2n)}{3 \cdot 5 \cdot \cdots \cdot (2n+1)}\right]^n \\ &> \left[\frac{2 \cdot 4 \cdot \cdots \cdot (2n)}{3 \cdot 5 \cdot \cdots \cdot (2n+1)}\right]^{n+1} = (k_n)^{n(n+1)}.\end{aligned}$$

So we have $k_{n+1} > k_n$ and then $p^*_{2n+2} < p^*_{2n}$. Since $p^*_{2n} > \frac{1}{2}$ is decreasing in n, the limit exists as $n \to \infty$. By the Stirling's approximation for the factorial, $n! = \sqrt{2\pi n}(n/e)^n (1 + o(1))$ and $(2n)! = \sqrt{4\pi n}(2n/e)^{2n} (1 + o(1))$, therefore $C^n_{2n} = \frac{2^{2n}}{\sqrt{\pi n}} (1 + o(1))$. So

$$\lim_{n\to\infty} [(2n+1)C^n_{2n}]^{\frac{1}{n}} = \lim_{n\to\infty} \exp\left[\frac{\ln(2n+1)}{n} + \ln 4 - \frac{\ln \pi}{2n} - \frac{\ln n}{2n}\right] = 4$$

and then $\lim_{n\to\infty} p^*_{2n} = \frac{1}{2}$. □

B.2 Proposition 8 and its proof

Proposition 8. *Let V^{pri}_{B+1} and V^{pri}_{B+3} be subject i's expected payoffs of observing one and three more additional signals after observing a black ball in the first stage. Assume that the sophisticated subject with belief error maximizes expected payoff. Then under Assumption 1, $V^{pri}_{B+3} > V^{pri}_{B+1}$.*

Proof. For notation simplicity, we omit the index i in the proof below. Let $v(p_x) \equiv P(\text{correct choice}_i \mid x) = \mathbb{E}\max(\tilde{p}^i_x, 1 - \tilde{p}^i_x)$. By Lemma 1, for $n = 1$ or 3, $V^{pri}_{B+n} = V^{(n)}_1 \tilde{p}_B + V^{(n)}_2 (1 - \tilde{p}_B)$, where

$$V^{(1)}_1 = pv(p_{BB}) + qv(p_{BW}) = pv\left(\frac{p^2}{p^2+q^2}\right) + qv\left(\frac{1}{2}\right),$$

$$V^{(1)}_2 = qv(p_{BB}) + pv(p_{BW}) = qv\left(\frac{p^2}{p^2+q^2}\right) + pv\left(\frac{1}{2}\right),$$

$$
\begin{aligned}
V_1^{(3)} &= p^3 v(p_{4B}) + 3p^2 q v(p_{3B1W}) + 3pq^2 v(p_{2B2W}) + q^3 v(p_{1B3W}) \\
&= p^3 v\left(\frac{p^4}{p^4+q^4}\right) + 3p^2 q v\left(\frac{p^2}{p^2+q^2}\right) + 3pq^2 v\left(\frac{1}{2}\right) + q^3 v\left(\frac{q^2}{p^2+q^2}\right), \\
V_2^{(3)} &= q^3 v(p_{4B}) + 3pq^2 v(p_{3B1W}) + 3p^2 q v(p_{2B2W}) + p^3 v(p_{1B3W}) \\
&= q^3 v\left(\frac{p^4}{p^4+q^4}\right) + 3pq^2 v\left(\frac{p^2}{p^2+q^2}\right) + 3p^2 q v\left(\frac{1}{2}\right) + p^3 v\left(\frac{q^2}{p^2+q^2}\right).
\end{aligned}
$$

Since $v(\cdot)$ is symmetric about $\frac{1}{2}$ and is monotone on $\left[\frac{1}{2}, 1\right]$ by Proposition 3, we have that for $\frac{1}{2} \leqslant p \leqslant 1$,

$$
\begin{aligned}
V_1^{(3)} &> p^3 v\left(\frac{p^2}{p^2+q^2}\right) + 3p^2 q\left(\frac{p^2}{p^2+q^2}\right) + 3pq^2 v\left(\frac{1}{2}\right) + q^3 v\left(\frac{p^2}{p^2+q^2}\right) \\
&= (1-3pq^2) v\left(\frac{p^2}{p^2+q^2}\right) + 3pq^2 v\left(\frac{1}{2}\right).
\end{aligned}
$$

Because $pq \leqslant \frac{1}{4}$ for any $\frac{1}{2} \leqslant p \leqslant 1$, we have $1 - 3pq \geqslant 1 - \frac{3}{4} > 0$ and therefore

$$
V_1^{(3)} - V_1^{(1)} > (1-3pq) q\left[v\left(\frac{p^2}{p^2+q^2}\right) - v\left(\frac{1}{2}\right)\right] > 0.
$$

Similarly, for $\frac{1}{2} \leqslant p \leqslant 1$,

$$
\begin{aligned}
V_2^{(3)} - V_2^{(1)} &> q^3 v\left(\frac{p^2}{p^2+q^2}\right) + 3pq^2 v\left(\frac{p^2}{p^2+q^2}\right) + 3p^2 q v\left(\frac{1}{2}\right) \\
&\qquad + p^3 v\left(\frac{p^2}{p^2+q^2}\right) - q v\left(\frac{p^2}{p^2+q^2}\right) - p v\left(\frac{1}{2}\right) \\
&= (1-3pq) p\left[v\left(\frac{p^2}{p^2+q^2}\right) - v\left(\frac{1}{2}\right)\right] > 0.
\end{aligned}
$$

This implies that $V_{B+3}^{pri} > V_{B+1}^{pri}$ for any any realization of $\tilde{p}_B$. □

B.3 Derive the Likelihood Function

It is simpler to derive the probability, $g_j(d_j^i \mid \gamma^i, \theta^i)$, that subject i chooses d_j^i after he observes a signal set (including the additional signals he buys), say x, in the j-th round. Note that this probability depends only on her belief after she observes the additional signals, so it will depends on the realizations of γ^i and θ^i but not α^i.

Given the signal set x, the subject will choose urn 1 if her belief about urn 1 $\tilde{p}_x^i > \frac{1}{2}$. By assumption, $\tilde{p}_x^i \sim \text{Beta}\left(\frac{p_x}{\gamma^i}, \frac{1-p_x}{\gamma^i}\right)$, subject i chooses urn 1 after observing signal set

x with probability $P\left(\tilde{p}^i_x > \frac{1}{2}\right) = 1 - \mathrm{I}_{\frac{1}{2}}\left(\frac{p_x}{\gamma^i}, \frac{1-p_x}{\gamma^i}\right)$, and chooses urn 2 with probability $\mathrm{I}_{\frac{1}{2}}\left(\frac{p_x}{\gamma^i}, \frac{1-p_x}{\gamma^i}\right)$. Therefore,

$$g_j(d^i_j \mid \gamma^i, \theta^i) = \begin{cases} 1 - \mathrm{I}_{\frac{1}{2}}\left(\frac{p_x}{\gamma^i}, \frac{1-p_x}{\gamma^i}\right) & \text{if } d^i_j = 1, \\ \mathrm{I}_{\frac{1}{2}}\left(\frac{p_x}{\gamma^i}, \frac{1-p_x}{\gamma^i}\right) & \text{if } d^i_j = 2. \end{cases}$$

If the first ball observed by subject i is black, we have calculated the posteriors p_{BB}, p_{BW}, p_{4B}, p_{3B1W}, ...(for private information setting) and p_{BC_1}, p_{BC_2}, p_{B3C_1}, $p_{B2C_11C_2}$, ...(for social information setting) in the proof of Lemma 1. If the first ball observed is white, the corresponding posteriors p_x can be calculated by[2]

$$p_{WW} = 1 - p_{BB}, \quad p_{4W} = 1 - p_{4B}, \quad p_{WC_1} = 1 - p_{BC_2}, \quad p_{WC_2} = 1 - p_{BC_1},$$
$$p_{W3C_1} = 1 - p_{B3C_2}, \quad p_{W2C_11C_2} = 1 - p_{B1C_12C_2}, \quad p_{W1C_12C_2} = 1 - p_{B2C_11C_2}, \quad p_{W3C_2} = 1 - p_{B3C_1}.$$

Now we derive the probability $f_j(c^i_j, b^i_j \mid \gamma^i, \theta^i, \alpha^i)$ in (1). By symmetry, the color of the first ball does not affect the bidding strategy, so we can compute the probability as if the first ball is black, and we can recover the value (or the range) of realized belief $\tilde{p}^i_B$ using the subject's choice c^i and bid b^i according to Proposition 7.

Because $1 - V_2 > 0$ and $V_2 < 1$, by Proposition 7, there is always a positive probability that a subject bids 0 regardless of the value of her α^i. Because the bidding function is non-decreasing in $\tilde{p}^i_B$ on $\left[0, \frac{1}{2}\right]$ and non-increasing in $\tilde{p}^i_B$ on $\left[\frac{1}{2}, 1\right]$, the subject will make the highest bid when $\tilde{p}^i_B = \frac{1}{2}$, which is equal to $b\left(\frac{1}{2}\right) = \frac{V_1+V_2-1}{2}(r + \alpha^i)$. This means that, for any given b, only those subjects with

$$\alpha^i \geqslant \frac{2b}{V_1 + V_2 - 1} - r$$

would possibly bid such a reservation price. As shown in Proposition 7, a subject may bid w_0 only if $\alpha^i \geqslant \frac{2w_0}{V_1+V_2-1} - r$. When subject i bids for multiple rounds, she forms a realized posterior belief in each round. Nevertheless, her α^i remains the same according to the present model. In such a case, the possible value of α^i will be restricted to

$$\alpha^i \geqslant \max\left(\frac{2b^i_1}{V_{11} + V_{21} - 1} - r, \ldots, \frac{2b^i_j}{V_{1J} + V_{2J} - 1} - r, 0\right) \equiv \hat{\alpha}^i \tag{1}$$

where V_{1j} and V_{2j} are the corresponding values of V_1 and V_2 in the j-th round. Then the

[2] In private information setting the order of the signals in the signal set is irrelevant.

likelihood (1) that is attributed to subject i becomes

$$L_i(\bar{\gamma},\bar{\theta},\bar{\alpha}) = \iint_{\mathbb{R}^2_+} \left\{ \int_{\hat{\alpha}^i}^{\infty} \left[\prod_{j=1}^{J} f_j(c^i_j, b^i_j \mid \gamma^i, \theta^i, \alpha^i) \prod_{j\in\mathcal{J}_i} g_j(d^i_j \mid \gamma^i, \theta^i) \right] \phi(\gamma^i, \theta^i, \alpha^i)\, \mathrm{d}\alpha^i \right\} \mathrm{d}\gamma^i\, \mathrm{d}\theta^i. \tag{2}$$

where $f_j(c^i_j, b^i_j \mid \gamma^i, \theta^i, \alpha^i)$ is determined as follows.

Case 1. If in the j-th round, the subject's choice is not consistent with the color of the first ball (i.e. $c^i_j = 2$ when the first ball is black or $c^i_j = 1$ when the first ball is white): In this case, $\tilde{p}^i_B \leqslant \frac{1}{2}$ and $V^t_{B+n} - \max(\tilde{p}^i_B, 1-\tilde{p}^i_B) = (1+V_1-V_2)\tilde{p}^i_B + V_2 - 1$.

(i) If $b^i_j = 0$, then it must be that $\tilde{p}^i_B \in \left[0, \frac{1-V_{2j}}{1+V_{1j}-V_{2j}}\right]$. So

$$f_j(c^i_j, b^i_j \mid \gamma^i, \theta^i, \alpha^i) = P\left(\tilde{p}^i_B \leqslant \frac{1-V_{2j}}{1+V_{1j}-V_{2j}}\right) = \mathrm{I}_{\frac{1-V_{2j}}{1+V_{1j}-V_{2j}}}\left(\frac{p}{\gamma^i}, \frac{q}{\gamma^i}\right);$$

(ii) If $0 < b^i_j < w_0$, since $b^i_j = (r+\alpha^i)[(1+V_{1j}-V_{2j})\tilde{p}^i_B + V_{2j} - 1]$, the subject's realized posterior belief $\tilde{p}^i_x$ is uniquely backed out from the bid as

$$\tilde{p}^i_B = b^{-1}(b^i_j) = \frac{b^i_j}{(r+\alpha^i)(1+V_1-V_2)} + \frac{1-V_2}{1+V_1-V_2}.$$

As a monotone transformation of the realized belief, the bid has probability density equal to $f_{\tilde{p}^i_B}(b^{-1}(b^i_j)) \left|\mathrm{d}b^{-1}(b^i_j)/\mathrm{d}b\right|$ where $f_{\tilde{p}^i_B}$ is the Beta $\left(\frac{p}{\gamma^i}, \frac{q}{\gamma^i}\right)$ density of $\tilde{p}^i_B$. Therefore, as $\mathrm{d}b^{-1}(b^i \mid \alpha^i)/\mathrm{d}b = \frac{1}{(r+\alpha^i)(1+V_1-V_2)} > 0$,

$$\begin{aligned} f_j(c^i_j, b^i_j \mid \gamma^i, \theta^i, \alpha^i) = {} & \frac{1}{\mathrm{B}\left(\frac{p}{\gamma^i}, \frac{q}{\gamma^i}\right)} \left[\frac{1-V_{2j}+b^i_j/(r+\alpha^i)}{1+V_{1j}-V_{2j}}\right]^{\frac{p}{\gamma^i}-1} \\ & \times \left[\frac{V_{1j}-b^i_j/(r+\alpha^i)}{1+V_{1j}-V_{2j}}\right]^{\frac{1-p}{\gamma^i}-1} \frac{1}{(r+\alpha^i)(V_{1j}+1-V_{2j})}; \end{aligned}$$

(iii) If $b^i_j = w_0$, then $\tilde{p}^i_B \in \left[\frac{1-V_{2j}+\Delta}{1+V_{1j}-V_{2j}}, \frac{1}{2}\right]$ with $\Delta = w_0/(r+\alpha^i)$ and

$$f_j(c^i_j, b^i_j \mid \gamma^i, \theta^i, u^i_0) = P\left(\frac{1-V_{2j}+\Delta}{1+V_{1j}-V_{2j}} < \tilde{p}^i_B \leqslant \frac{1}{2}\right) = \mathrm{I}_{\frac{1}{2}}\left(\frac{p}{\gamma^i}, \frac{q}{\gamma^i}\right) - \mathrm{I}_{\frac{1-V_{2j}+\Delta}{1+V_{1j}-V_{2j}}}\left(\frac{p}{\gamma^i}, \frac{q}{\gamma^i}\right).$$

Case 2. If in the j-th round, the subject's choice is consistent with the color of the first ball (i.e. $c_j^i = 1$ when the first ball is black or $c_j^i = 2$ when the first ball is white): In this case, $\tilde{p}_B^i \geqslant \frac{1}{2}$ and $V_{B+n}^t - \max(\tilde{p}_B^i, 1 - \tilde{p}_B^i) = (V_{1j} - V_{2j} - 1)\tilde{p}_B^i + V_{2j}$.

(i) If $b_j^i = 0$, then $\tilde{p}_B^i \in \left[\frac{V_{2j}}{1+V_{2j}-V_{1j}}, 1\right]$. So

$$f_j(c_j^i, b_j^i \mid \gamma^i, \theta^i, u_0^i) = P\left(\tilde{p}_B^i \geqslant \frac{V_{2j}}{1 + V_{2j} - V_{1j}}\right) = 1 - \mathrm{I}_{\frac{V_{2j}}{1+V_{2j}-V_{1j}}}\left(\frac{p}{\gamma^i}, \frac{q}{\gamma^i}\right);$$

(ii) If $0 < b_j^i < w_0$, since $b_j^i = (r + u_0^i)[(V_{1j} - V_{2j} - 1)\tilde{p}_B^i + V_{2j}]$, the realized belief

$$\tilde{p}_B^i = b^{-1}(b_j^i) = -\frac{b_j^i}{(r + u_0^i)(1 + V_{2j} - V_{1j})} + \frac{V_{2j}}{1 + V_{2j} - V_{1j}}$$

and $\mathrm{d}b^{-1}(b_j^i \mid u_0^i)/\mathrm{d}b = -\frac{1}{(r+u_0^i)(1+V_2-V_1)} < 0$. Then,

$$f_j(c_j^i, b_j^i \mid \gamma^i, \theta^i, u_0^i) = \frac{1}{\mathrm{B}\left(\frac{p}{\gamma^i}, \frac{q}{\gamma^i}\right)} \left[\frac{V_{2j} - b_j^i/(r + u_0^i)}{1 + V_{2j} - V_{1j}}\right]^{\frac{p}{\gamma^i}-1} \times \left[\frac{1 - V_{1j} + b_j^i/(r + u_0^i)}{1 + V_{2j} - V_{1j}}\right]^{\frac{q}{\gamma^i}-1} \frac{1}{(r + u_0^i)(1 + V_{2j} - V_{1j})};$$

(iii) If $b_j^i = w_0$, then $\tilde{p}_B^i \in \left[\frac{1}{2}, \frac{V_{2j}-\Delta}{1+V_{2j}-V_{1j}}\right]$ and

$$f_j(c_j^i, b_j^i \mid \gamma^i, \theta^i, u_0^i) = P\left(\frac{1}{2} < \tilde{p}_B^i \leqslant \frac{V_{2j} - \Delta}{1 + V_{2j} - V_{1j}}\right) = \mathrm{I}_{\frac{V_{2j}-\Delta}{1+V_{2j}-V_{1j}}}\left(\frac{p}{\gamma^i}, \frac{q}{\gamma^i}\right) - \mathrm{I}_{\frac{1}{2}}\left(\frac{p}{\gamma^i}, \frac{q}{\gamma^i}\right).$$

C Experimental Instructions

(Translated from Chinese)

This is an experiment of economics in decision making and you will play 60 rounds.

In each round, a computer first selects either of urn 1 and urn 2 with an equal chance (See the table below) and all participants will use the selected urn in this round. Urn 1 of type $\frac{12}{20}$ includes twelve black balls and eight white balls, and urn 2 of this type contains twelve white balls and eight black balls. Urn 1 of type $\frac{15}{20}$ includes fifteen black balls and five white balls, and urn 2 of this type contains fifteen white balls and five black balls. Urn 1 of type $\frac{18}{20}$ includes eighteen black balls and two white balls, and urn 2 of this type contains eighteen white balls and two black balls.

Compositions of each urn

	Urn 1	Urn 2
Type $\frac{12}{20}$	12 black balls 8 white balls	8 black balls 12 white balls
Type $\frac{15}{20}$	15 black balls 5 white balls	5 black balls 15 white balls
Type $\frac{18}{20}$	18 black balls 2 white balls	2 black balls 18 white balls

It is a public information that the type of the selected urn in a round is revealed to all participants. However, the label of the selected urn (ie., urn 1 or urn 2) is not revealed to participants. Participants are expected to guess the label of the selected urn in a round based on the information they have.

Decision task in a decision round

The computer independently and randomly draws one ball from the selected urn in this round with replacement for each participant and informs the participant of the color of the ball. In other words, the computer randomly draws one ball from the selected urn and reveals its color to participant 1, then the ball is returned to the urn, and then the computer randomly draws a ball from the urn and reveals its color to participant 2, and then the same procedure is repeatedly conducted till for the last participant. Based on the revealed color of the ball, each participant is asked to make a first guess about the label of the selected urn in this round.

After the first guess, each participant is endowed with 300 tokens and is asked to how much she would like to pay for an additional information. Depending on the specific round, the additional information may belong to one of the following four categories: (1) the color of a ball that is randomly drawn from the selected urn; (2) the colors of three balls that are independently and randomly drawn from the selected urn (Specifically, a first ball is randomly drawn, its color is recorded, and the ball is returned to the urn; then a second ball is randomly drawn, its color is recorded, and the ball is returned to the urn; and then a third ball is randomly drawn, its color is recorded, and the ball is returned to the urn); (3) the first guess about the label of the selected urn made by another participant; (4) the first guesses about the label of the selected urn made by three other participants.

Specifically, a participant pays for the additional information according to the procedure below. She is firstly told of the category which the additional information belongs to. Then she is asked to state her willingness to pay, say b tokens, from $\{0, 1, 2, ..., 299, 300\}$.

The computer then chooses an ask price, say s tokens, where the ask price s is randomly and equally likely chosen from $\{0, 1, 2, ..., 299, 300\}$. If $b \geqslant s$, the participant is charged s tokens (collected from her endowment of 300 tokens) and is provided with the additional information , and then is asked to make a second guess about the label of the selected urn. In this case, her second guess is recorded as her final guess for the decision round. If $b < s$, the participant is not charged, is not provided with the additional information, and is not asked to make a second guess. In this case, her first guess is recorded as her final guess for the decision round.

At the end of a decision round, participants are told of the label of the used urn.[3]

Payoffs

When a participant completes the 60-round tasks, the computer randomly picks one round and the participant will be paid according to her final guess in that round: she earns 300 tokes for a right guess and 0 tokens for a wrong guess. The final guess is right if it is the same as the label of the selected urn; the guess is wrong if it is different from the label of the selected urn.

If a participant was not provided with the additional information in that round, she still keeps her endowment of 300 tokens. If a participant was provided with the additional information with the charge of s tokens in that round, she keeps part of her endowment, i.e., $300 - s$ tokens.

Finally, the total number of tokens a participant has is the sum of the number of tokens she earns in the paid round and the endowment that is still kept in that round. Every 10 tokens can be redeemed for one Chinese yuan.

Rules

If you have any question during the experiment, please raise your hand. You are neither allowed to talk with each other during the experiment nor allowed to leave without a permission from the investigator. Please remain seated till you are called at the end of the experiment.

[3]This paragraph only appears in the Experimental Instructions for feedback sessions.